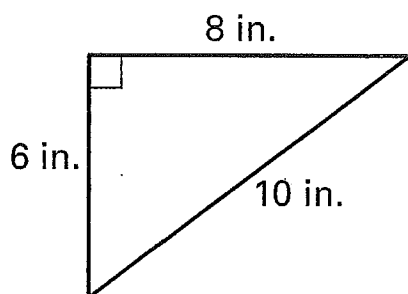


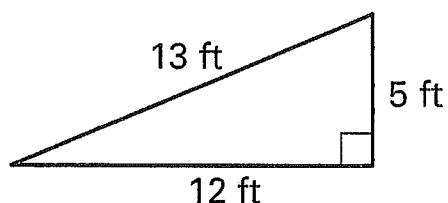
Section 12-5 The Pythagorean Theorem

Square the length of each side of the right triangle shown in the picture. Add the squares of the lengths of the two shorter sides. Tell how this sum compares to the square of the length of the longest side.

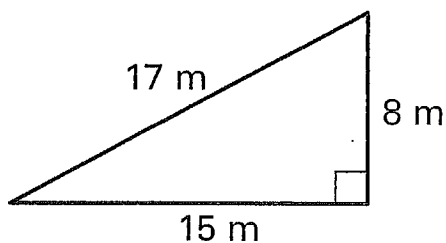
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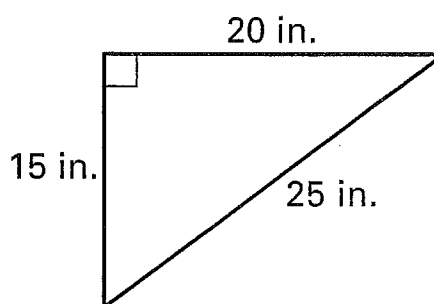
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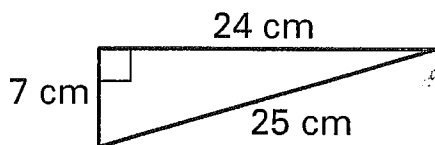
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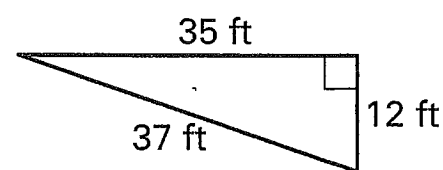
4.



5.



6.



7. Make a conjecture about the relationship between the lengths of the sides of a right triangle.

Math and History Application

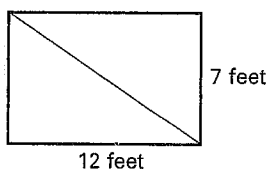
For use with page 744

HISTORY Pythagoras was born around 580 B.C. in Samos, Greece. He studied mathematics, music theory, and astronomy. Although the Pythagorean theorem is credited to Pythagoras, other cultures used the theorem many years before Pythagoras' time. The Chinese used the Pythagorean theorem to survey land, and the Egyptians used it to build pyramids, in each case, centuries before Pythagoras was born. Babylonian and Indian mathematicians used sets of three numbers where the sum of the squares of two is equal to the third. Lists of these triplets were found in the *Sulva Sultras*, which was written between 800 B.C. and 500 B.C. Trade was widespread between these areas so it is unknown where the theorem was first developed.

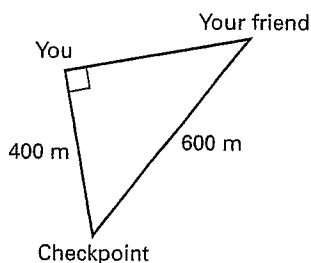
Pythagoras lived in Babylon for 20 years. During this time he studied and taught astronomy, mathematics, and astrology. He also traveled in Egypt and became acquainted with the mathematics of the area. These experiences probably led Pythagoras to record the theorem that bears his name.

MATH Pythagoras returned to Italy and set up a secret society devoted to exploring the mysteries of numbers. He studied the relationship between numbers and musical harmonies. He developed theories for perfect, triangular, and square numbers. The Pythagorean theorem is very useful and has many applications.

1. You are building a tool shed. The framing for the floor measures 12 feet by 7 feet. What must the diagonal measure be to ensure the floor is square? (Carpenters consider the framing square when the angles measure 90°).



2. You and a friend are participating in an orienteering competition. Your friend travels off course and is now 600 meters from the checkpoint. You are 400 meters from the checkpoint. How far are you from your friend?



Practice with Examples

For use with pages 738–744

GOAL

Use the Pythagorean theorem and its converse and use the Pythagorean theorem in real-life problems

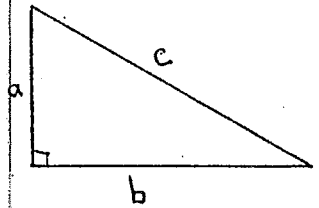
VOCABULARY

In a right triangle, the **hypotenuse** is the side opposite the right angle; the other two sides are the **legs**.

The **Pythagorean theorem** states that if a triangle is a right triangle, then the sum of the squares of the lengths of the legs a and b equals the square of the length of the hypotenuse c , or $a^2 + b^2 = c^2$.

In a statement of the form “If p , then q ,” p is the **hypothesis** and q is the **conclusion**. The **converse** of the statement “If p , then q ” is the related statement “If q , then p .”

The **converse of the Pythagorean theorem** states that if a triangle has side lengths a , b , and c such that $a^2 + b^2 = c^2$, then the triangle is a right triangle.



Pythagorean Theorem

$$a^2 + b^2 = c^2$$

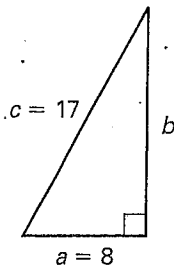
$$a^2 + b^2 = c^2$$

$$a^2 + b^2 = c^2$$

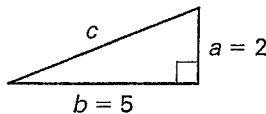
Exercises

Use the Pythagorean theorem to find the missing length of the right triangle.

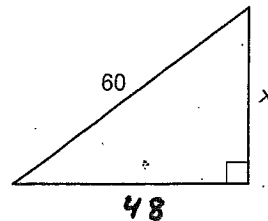
1.



2.



3.



Determine whether the given lengths are sides of a right triangle.

- a. 2.5, 6, 6.5 b. 10, 24, 25

SOLUTION

Use the converse of the Pythagorean theorem.

- a. The lengths are sides of a right triangle because

$$2.5^2 + 6^2 = 6.25 + 36 = 42.25 = 6.5^2.$$

- b. The lengths are not sides of a right triangle because

$$10^2 + 24^2 = 100 + 576 = 676 \neq 25^2.$$

Exercises for Example 2

Determine whether the given lengths are sides of a right triangle.

4. 8, 15, 17

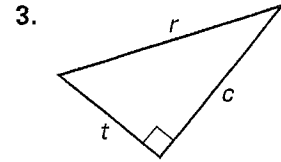
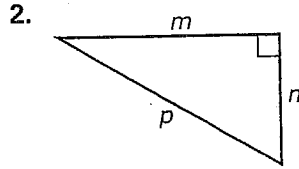
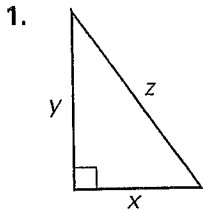
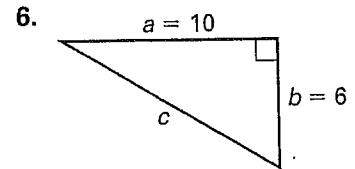
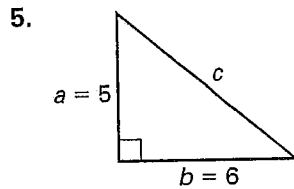
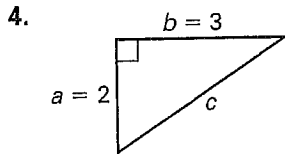
5. 3, 6, 7

6. 9, 40, 41

Practice A

For use with pages 738–744

Name the legs and the hypotenuse of the right triangle.

Find the missing length of the right triangle if a and b are the lengths of the legs and c is the length of the hypotenuse.

7. $a = 4, b = 6$

8. $a = 7, b = 3$

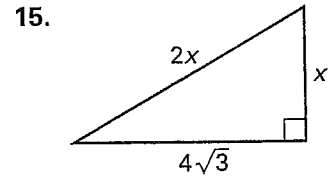
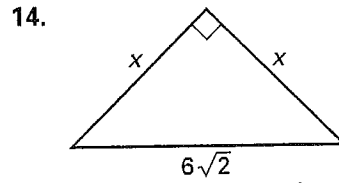
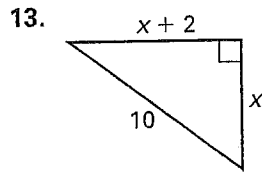
9. $a = 6, b = 6$

10. $a = 12, b = 9$

11. $a = 6, b = 8$

12. $a = 5, b = 12$

Find each missing length.



Determine whether the given lengths are sides of a right triangle. Explain your reasoning.

16. 2, 2, 4

17. 6, 9, 12

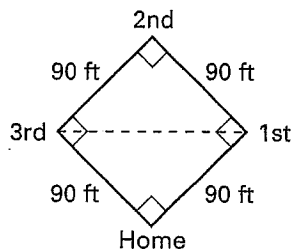
18. 10, 15, 20

19. 10, 24, 26

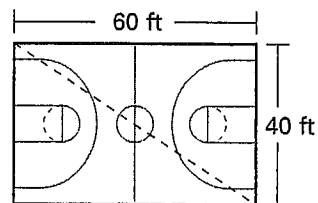
20. 5, 5, $5\sqrt{2}$

21. 30, 40, 50

22. **Baseball** The infield of a baseball field is a square. The distance between consecutive bases is 90 feet. How far is it from first base to third base?



23. **Basketball** A basketball court is a rectangle. If the court measures 60 feet by 40 feet, what is the length of the diagonal from one corner of the court to the opposite corner?

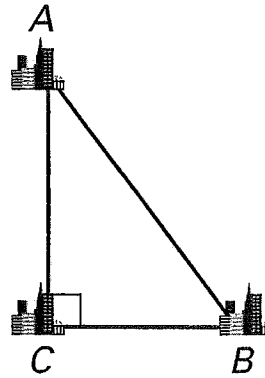


Section 12-6 The Distance and Midpoint Formulas
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Use the figure at the right.

The distance between city A and city C is 12 miles and the distance between city B and city C is 9 miles.

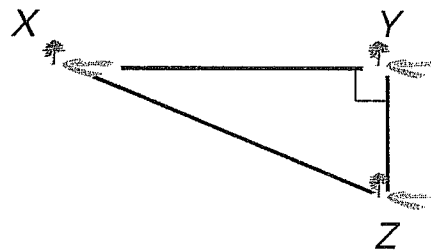
1. Suggest a way to find the distance between city A and city B .
2. Use your suggestion to find the distance.



Use the figure at the right.

The distance between park Z and park Y is 5 miles and the distance between park Y and park X is 12 miles.

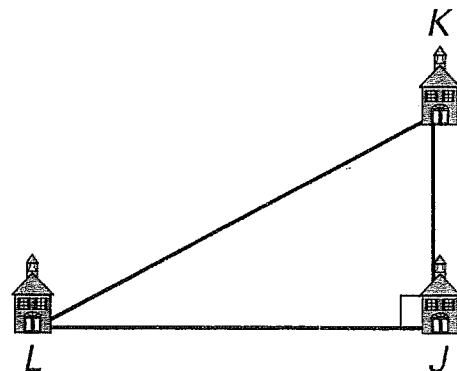
3. Suggest a way to find the distance between park X and park Z .
4. Use your suggestion to find the distance.



Use the figure at the right.

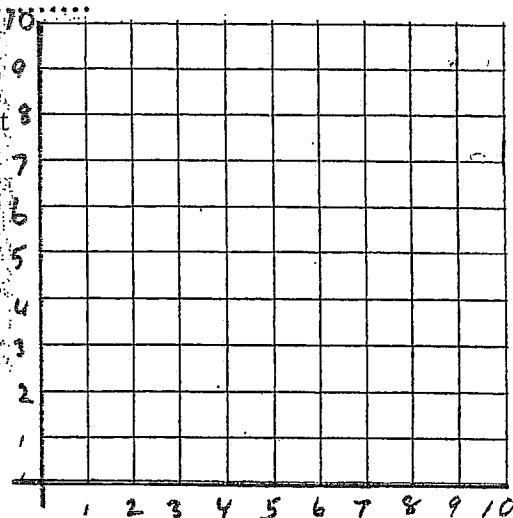
The distance between school J and school K is 8 miles and the distance between school J and school L is 15 miles.

5. Suggest a way to find the distance between school K and school L .
6. Use your suggestion to find the distance.



▶ ACTIVITYDeveloping
Concepts**Investigating Distance**

- 1 Plot $A(2, 1)$ and $B(6, 4)$ on graph paper. Then draw a right triangle that has \overline{AB} as its hypotenuse.
- 2 Label the coordinates of the vertex C .
- 3 Find the lengths of the legs of $\triangle ABC$.
- 4 Use the Pythagorean theorem to find AB .
- 5 Check the distance by actual measurement.

**THE DISTANCE FORMULA**

The distance d between the points (x_1, y_1) and (x_2, y_2) is

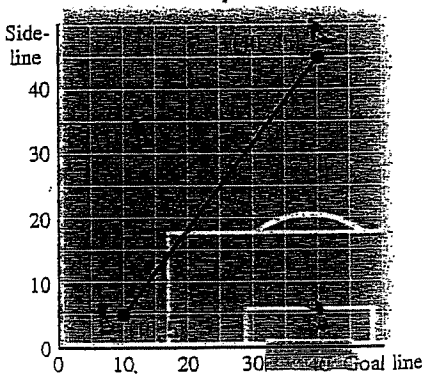
$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

EXAMPLE 1 *Finding the Distance Between Two Points*

Find the distance between $(1, 4)$ and $(-2, 3)$.

**EXAMPLE 2** *Applying the Distance Formula*

A player kicks a soccer ball from a position that is 10 yards from a sideline and 5 yards from a goal line. The ball lands at a position that is 45 yards from the same goal line and 40 yards from the same sideline. How far was the ball kicked?



GOAL 2 FINDING THE MIDPOINT BETWEEN TWO POINTS

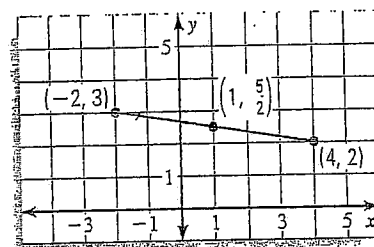
The **midpoint** of a line segment is the point on the segment that is equidistant from its end-points. The *midpoint between two points* is the midpoint of the line segment connecting them.

THE MIDPOINT FORMULA

The midpoint between (x_1, y_1) and (x_2, y_2) is $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.

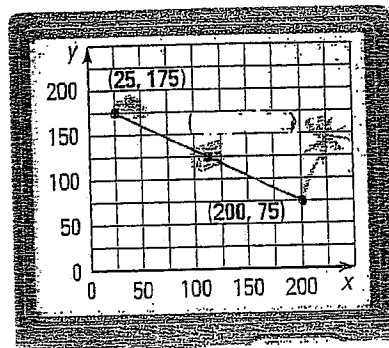
EXAMPLE 4 Finding the Midpoint Between Two Points

Find the midpoint between $(-2, 3)$ and $(4, 2)$. Use a graph to check the result.



EXAMPLE 5 Applying the Midpoint Formula

You are using computer software to design a video game. You want to place a buried treasure chest halfway between the center of the base of a palm tree and the corner of a large boulder. Find where you should place the treasure chest.



GUIDED PRACTICE

Vocabulary Check ✓

1. What is meant by the *midpoint* between two points?

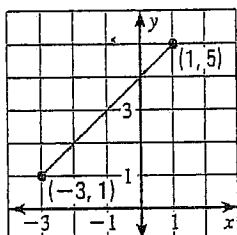
Concept Check ✓

2. Explain how you can use the Pythagorean theorem to find the distance between any two points in a coordinate plane.

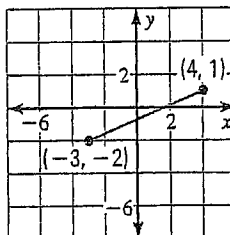
Skill Check ✓

Use the coordinate plane to estimate the distance between the two points. Then use the distance formula to find the distance between the points. Round the result to the nearest hundredth.

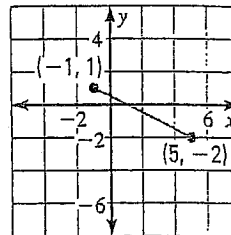
3. $(1, 5), (-3, 1)$



4. $(-3, -2), (4, 1)$



5. $(5, -2), (-1, 1)$




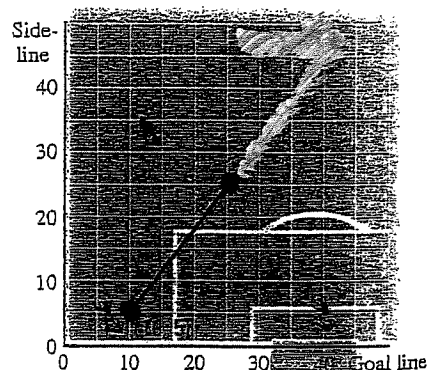
Find the midpoint between the two points.

10. $(4, 4), (-1, 2)$

11. $(6, 2), (2, -3)$

12. $(-5, 3), (-3, -3)$

13.  **SOCCER** Suppose the soccer ball in Example 2 lands in a position that is 25 yards from the same goal line and 25 yards from the same sideline. How far was the ball kicked?



12-6 The Distance and Midpoint Formulas

Write the distance formula:

Find the distance between $(3, -1)$ and $(0, 3)$.

Write the midpoint formula:

Find the midpoint between $(-1, 1)$ and $(-4, -4)$.

Algebra

Name _____

Review for Ch. 11 & Ch. 12

Date _____ Hour _____

SHOW ALL WORK**Section 11.1 Ratio and Proportion
Simplify.**

1. $\frac{16}{4} = \frac{12}{x}$

2. $\frac{5}{8} = \frac{x}{9}$

3. $\frac{x+4}{3} = \frac{x}{5}$

4. $\frac{-2}{x-7} = \frac{x}{5}$

5. $\frac{x-3}{x} = \frac{x}{x+6}$

6. $\frac{2}{6x+1} = \frac{2x}{1}$

**Section 11.2 Percents
Solve.**

7. What is 25% of 150?

8. 85% of 400 is what number?

9. 18 is what percent of 60?

10. 16% of what number is 8?

11. \$240 is what percent of \$50?

12. 42 feet is 50% of what length?

**Section 11.4 Simplifying Rational Expressions
Simplify the expression if possible.**

13. $\frac{5x}{20}$

14. $\frac{-30x^4}{18x^2}$

Simplify the expression if possible.

15. $\frac{8x}{5x+x^2}$

16. $\frac{x^2+x-20}{x^2+2x-15}$

17. $\frac{2x^2+11x-6}{x+6}$

18. $\frac{x^3+9x^2+14x}{x^2-4}$

Section 12.5 The Pythagorean Theorem

$$a^2 + b^2 = c^2$$

Find the missing length of the right triangle.

19. $a = 7, b = 24$

20. $a = 5, c = 13$

21. $b = 15, c = 17$

22. $a = 12, b = 35$

Determine whether the given lengths are sides of a right triangle.

23. 9,12,15

24. 8,10,13

Section 12.6 The Distance and Midpoint Formulas

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Find the distance between the two points.

25. $(-2, -2), (2, 2)$

26. $(3, -6), (8, 6)$

27. $(-1, -4), (-2, -5)$

28. $(4, -1), (5, 8)$

Find the midpoint between the two points.

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

29. $(4, 6), (2, 8)$

30. $(8, -5), (4, -3)$