Algebra/Factoring (1)

Garrity

GCF

Name ____________________

Date ____________________

Hour ____________________

Name the monomial with the greatest numerical coefficient and the greatest degree in each variable that is a factor of both monomials in each pair.

Sample: $9a^3b;\ 12a^2b$

Solution: $3a^2b$

1) $16;\ 24x$

2) $12;\ 18y^2$

3) $5xz^2;\ 7x^2z^3$

4) $30a^3b^2;\ 45a^2b$

5) $20r^3s^2;\ 100rs^3$

6) $28k^5t;\ 35k^2t^2$

7) $7r^5s^2;\ 15t^3$

8) $16x^5y;\ 64x^2y^5$

9) $154a^2b;\ 132ab^4$

10) $78x^3y^2;\ 52x^2y^4$

11) $126x^2yz^3;\ 105x^3yz^2$

12) $108a^3b^2c;\ 144a^2bc^3$

13) $176r^2s^3;\ 208s^2t^3$

14) $132m^4n^3;\ 156n^2p^3$

Give the second factor for each monomial.

15) $6a^2b = 6a(\underline{\ ?\ })$

16) $24r^2s = (12rs)(\underline{\ ?\ })$

17) $-15x^3y^2 = (-5x^2y^2)(\underline{\ ?\ })$

18) $-32p^3q^4 = (16p^3q^2)(\underline{\ ?\ })$

19) $42u^3v^2w = (-6u^2vw)(\underline{\ ?\ })$

20) $51x^4y^2z^3 = (-17x^3y^2z^2)(\underline{\ ?\ })$

21) $72r^3s^5t^2 = (18r^3s^4t^2)(\underline{\ ?\ })$

22) $102a^3b^2c = (-17a^3b)(\underline{\ ?\ })$

Algebra/Factoring (2)

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Date ____________________

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Write in factored form. Check by multiplying your factors (distributive property).

Sample: $15t^3 - 5t$

Solution: $15t^3 - 5t = 5t(3t^2 - 1)$

1) $4x^2 - 8$

2) $6 + 9y$

3) $4a^2 + 5a$

4) $6b^2 + 7b$

5) $10z^3 - 5z^2$

6) $12n^2 + 24n^3$

7) $6a^2b + 3ab^2$

8) $7rs^3 - 14r^2s$

9) $3p^2 + 3p - 9$

10) $4 - 6q + 10q^2$

11) $5n^2 - 15n + 20$

12) $6 - 3p + 18p^2$

13) $a^3x^3 + a^2x^2 - ax$

14) $b^4y^2 + b^3y^3 - b^2y^4$

15) $4b^2y^2 - 8b^2y + 24b^2$

16) $35k^2 - 42k^2t + 14k^2t^2$

17) $15x^2y - 30xy + 35xy^2$

18) $-18u^3v^2 + 12u^2v^2 - 6uv^2$

19) $9x^5y^2 - 6x^4y^3 + 3x^3y^4$

20) $-12x^3y^3 + 18x^2y^4 + 27xy^5$

Write each expression in factored form, and check.

Sample: $y^2(y + 3) + 2(y + 3)$

Solution: $y^2(y + 3) + 2(y + 3) = (y^2 + 2)(y + 3)$

21) $n(n - 1) + 3(n - 1)$

22) $x(2x + 3) + 2(2x + 3)$

23) $(3a - b)b + (3a - b)a$

24) $(c + 3d)(2c) + (c + 3d)(3d)$

25) $t^2(y + 5) - 5(y + 5)$

26) $k^2(t + 1) + 2k(t + 1)$

27) $n^2(2n + 1) + (2n + 1)$

28) $2m^2(3m + 1) + (3m + 1)$

29) $a^2(a - b) + b(a - b)$

30) $2n(n^2 + 1) + 3(n^2 + 1)$

31) $5c(a^3 + b) - (a^3 + b)$

32) $m(m + 2n) - n(m + 2n)$

Sample: $x^2 + x + xy + y$

Solution: $x^2 + x + xy + y = x(x + 1) + y(x + 1)$

$= (x + y)(x + 1)$

33) $n^2 + 2n + np + 2p$

34) $a^2 - 3a + ay - 3y$

Algebra/Factoring (3)

Garrity

Binomials

Name ______________________

Date ______________________

Hour ______________________

Multiplying the Sum and Difference of Two Numbers

Certain products occur so often that you should recognize them at sight.
Study each of the three examples below:

$$\begin{array}{rr} y & +2 \\ y & -2 \\ \hline y^2 & +2y \\ & -2y-4 \\ \hline y^2 & -4 \end{array} \qquad \begin{array}{rr} 3a & -2b \\ 3a & +2b \\ \hline 9a^2 & -6ab \\ & 6ab-4b^2 \\ \hline 9a^2 & -4b^2 \end{array} \qquad \begin{array}{rr} a & +b \\ a & -b \\ \hline a^2 & +ab \\ & -ab-b^2 \\ \hline a^2 & -b^2 \end{array}$$

Express each product as a polynomial.

Sample: $(x-1)(x+1)$

Solution: x^2-1

1) $(y+2)(y-2)$

2) $(z+3)(z-3)$

3) $(x-y)(x+y)$

4) $(p-q)(p+q)$

5) $(t + 6)(t - 6)$

6) $(n - 8)(n + 8)$

7) $(2a - 1)(2a + 1)$

8) $(3b - 1)(3b + 1)$

9) $(y^2 - 5)(y^2 + 5)$

10) $(z^3 + 9)(z^3 - 9)$

11) $(3r + \frac{1}{2})(3r - \frac{1}{2})$

12) $(5k + \frac{2}{3})(5k - \frac{2}{3})$

Algebra/Factoring (4)

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Factoring the Difference of Two Squares

Name ____________________

Date ____________________

Hour ____________________

By the symmetric property of equality, the relation $(a+b)(a-b) = a^2 - b^2$ is reversible: $a^2 - b^2 = (a+b)(a-b)$

Samples:

$$x^2 - 9 = (x+3)(x-3)$$
$$16y^2 - x^2 = (4y+x)(4y-x)$$
$$25c^2 - 49d^2 = (5c+7d)(5c-7d)$$
$$-u^2v^2 + x^2y^2 = x^2y^2 - u^2v^2 = (xy+uv)(xy-uv)$$

Factor.

1) $t^2 - 9$

2) $k^2 - 4$

3) $4m^2 - 1$

4) $9r^2 - 1$

5) $t^2 - u^2$

6) $x^2 - 16y^2$

7) $25v^2 - 49$

8) $64 - 25r^2$

9) $81t^2 - 100s^2$

10) $121a^2 - 144b^2$

11) $a^2b^4 - c^2$

12) $d^2 - f^2g^4$

13) $4y^2 - 1$

14) $16t^2 - 9$

15) $25x^2 - 36y^2$

16) $49m^2 - 64$

17) $4r^2 - 64$

18) $9p^2 - 81$

19) $x^2 - 169$

20) $144 - n^2$

21) $225a^2 - b^2$

22) $4x^2 - 625$

Algebra/Factoring (5) Name ____________________

Garrity Date ____________________

Quadratic Trinomials-Squaring a Binomial Hour ____________________

The binomial $a + b$ is squared at the right by the usual method of multiplication. Notice how each term in the product is obtained.

$$\begin{array}{rrrrr} & a & + & b & \\ & a & + & b & \\ \hline & a^2 & + & ab & \\ & & & ab & + b^2 \\ \hline & a^2 & + & 2ab & + b^2 \end{array}$$

1. Square the first term in the binomial. → a^2

2. Double the product of the two terms. → $2ab$

3. Square the second term in the binomial. → b^2

Now examine the square of a binomial difference. The binomial $a - b$ is squared by multiplication at the right. Again, notice how each term is obtained.

$$\begin{array}{rrrrr} & a & - & b & \\ & a & - & b & \\ \hline & a^2 & - & ab & \\ & & - & ab & + b^2 \\ \hline & a^2 & - & 2ab & + b^2 \end{array}$$

1. Square the first term in the binomial. → a^2

2. Double the product of the two terms. → $2ab$

3. Square the second term in the binomial. → b^2

Whenever you square a binomial, the product is a **trinomial square**, whose terms show this pattern:

$$(a + b)^2 = a^2 + 2ab + b^2$$
$$(a - b)^2 = a^2 - 2ab + b^2$$

Knowing these relationships, you can write the square of a binomial without performing long multiplication.

Samples:

1. $(c + 1)^2 = c^2 + 2c + 1$
2. $(d - 1)^2 = d^2 - 2d + 1$
3. $(2x + 5)^2 = 4x^2 + 20x + 25$
4. $(6z^2 - w^3)^2 = 36z^4 - 12z^2w^3 + w^6$
5. $(-r^2s + t^3)^2 = (t^3 - r^2s)^2 = t^6 - 2t^3r^2s + r^4s^2$

Write each power as a trinomial.

1) $(p + 7)^2$

2) $(q - 8)^2$

3) $(2x - 1)^2$

4) $(3y + 1)^2$

5) $(4t + 3)^2$

6) $(3s - 2)^2$

7) $(6r + 5)^2$

8) $(7k - 3)^2$

9) $(2x - 3y)^2$

10) $(5z + 2u)^2$

11) $(xy - 1)^2$

12) $(2 + rs)^2$

Algebra/Factoring (6)
Garrity
Factoring a Trinomial Square

Name ____________________
Date ____________________
Hour ____________________

To factor a trinomial square, you reverse the equations you use in squaring a binomial.

$$a^2 + 2ab + b^2 = (a + b)^2$$
$$a^2 - 2ab + b^2 = (a - b)^2$$

Before you use one of these equations as a rule for factoring, you must be sure that the expression to be factored is a trinomial square. Arrange the terms of the trinomial with exponents in descending order, and then examine each term to see how it may have been obtained. For example,

$$y^2 + 10y + 25$$

Is the first term a square? Yes, y^2 is the square of y.
Is the third term a square? Yes, 25 is 5^2.
Is the middle term (neglecting the sign) double the product of 5 and y?
Yes, $10y = 2\,(y)\,(5)$.
Therefore, this trinomial is a square. Since all its terms are positive, it is the square of a sum; thus,

$$y^2 + 10y + 25 = (y+5)^2$$

Example:

Factor $81r^2 - 198rs + 121s^2$.

Solution: $81r^2 - 198rs + 121s^2$

$= (9r)^2 - 2(9r)(11s) + (11s)^2$

Thus, $81r^2 - 198rs + 121s^2 = (9r - 11s)^2$. Answer.

Check: $(9r - 11s)^2 = 81r^2 - 198rs + 121s^2$ ✓

Factor.

1) $n^2 - 2n + 1$

2) $m^2 + 2m + 1$

3) $k^2 - 6k + 9$

4) $t^2 - 8t + 16$

5) $r^2 + 10r + 25$

6) $s^2 + 12s + 36$

7) $4p^2 - 4p + 1$

8) $9q^2 + 6q + 1$

9) $16c^2 + 8c + 1$

10) $25d^2 - 10d + 1$

11) $4x^2 - 4xy + y^2$

12) $9u^2 + 6uv + v^2$

13) $1 - 4t + 4t^2$

14) $25 + 10k + k^2$

15) $64y^2 - 16yz + z^2$

16) $81k^2 + 18kt + t^2$

17) $9 + 12p + 4p^2$

18) $36 - 60q + 25q^2$

19) $4x^2y^2 - 12xyz + 9z^2$

20) $16t^2 + 24tuv + 9u^2v^2$

21) $x^4 + 2x^2 + 1$

22) $y^4 + 10y^2 + 25$

23) $25y^4 - 10y^2x + x^2$

24) $z^2 + 18zab + 81a^2b^2$