Algebra **2A**

Name ________________________ 4th

Date __________________ Hr ____

Section 3.4

GOAL **Solve systems of equations in three variables.**

Vocabulary

A **linear equation in three variables** x, y, and z is an equation of the form $ax + by + cz = d$ where a, b, and c are not all zero.

An example of a **system of three linear equations** in three variables:

$x + 2y + z = 3$ Equation 1

$2x + y + z = 4$ Equation 2

$x - y - z = 2$ Equation 3

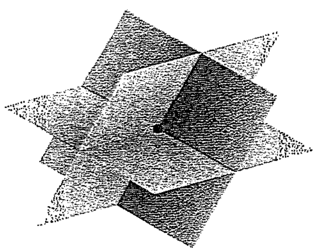
A **solution of a system with three variables** is an **ordered triple** (x, y, z) whose coordinates make each equation true.

A **solution** of such a system is an **ordered triple** (x, y, z) whose coordinates make each equation true.

The graph of a linear equation in three variables is a plane in three-dimensional space. The graphs of three such equations that form a system are three planes whose intersection determines the number of solutions of the system, as shown in the diagrams below.

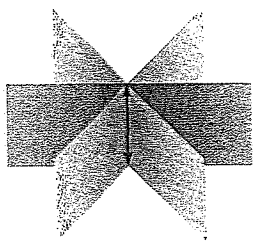
Exactly one solution

The planes intersect in a single point.



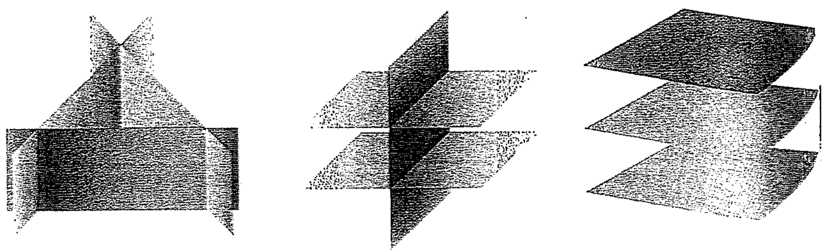
Infinitely many solutions

The planes intersect in a line or are the same plane.



No solution

The planes have no common point of intersection.



Assignment 3-4 Pages 182-183, #3, 9, 14, 15, 17, 23 (use any method to solve)

EXAMPLE 1 Use the elimination method

Solve the system.

$2x + 3y - z = 13$ **Equation 1**

$3x + y - 3z = 11$ **Equation 2**

$x - y + z = 3$ **Equation 3**

STEP 1 Rewrite the system as a linear system in two variables.

$$\begin{aligned} 2x + 3y - z &= 13 \\ 3x - 3y + 3z &= 9 \\ \hline 5x \quad + 2z &= 22 \end{aligned}$$

Add 3 times the third equation to the first equation. New Equation 1

$$\begin{aligned} 3x + y - 3z &= 11 \\ x - y + z &= 3 \\ \hline 4x \quad - 2z &= 14 \end{aligned}$$

Add the second and third equations. New Equation 2

STEP 2 Solve the new linear system for both of its variables.

$$\begin{aligned} 5x + 2z &= 22 \\ 4x - 2z &= 14 \\ \hline 9x &= 36 \end{aligned}$$

Add new Equation 1 and new Equation 2.

$x = 4$ Solve for x.

$z = 1$ Substitute into new Equation 1 or 2 to find z.

STEP 3 Substitute $x = 4$ and $z = 1$ into an original equation and solve for y.

$x - y + z = 3$ Write original Equation 3.

$4 - y + 1 = 3$ Substitute 4 for x and 1 for z.

$y = 2$ Solve for y.

The solution is $x = 4$, $y = 2$, and $z = 1$ or the ordered triple (4, 2, 1).

EXAMPLE 2 Solve a three-variable system with no solution

Solve the system.

$2x - 2y + 2z = 9$ **Equation 1**

$x - y + z = 5$ **Equation 2**

$3x + y + 2z = 4$ **Equation 3**

When you multiply the second equation by -2 and add the result to the first equation, you obtain a false equation.

$$\begin{aligned} 2x - 2y + 2z &= 9 \\ -2x + 2y - 2z &= -10 \\ 0 &= -1 \end{aligned}$$

New Equation 1

Because you obtain a false equation, the original system has no solution.

Exercises

Solve the system.

1. $2x + y + z = 5$
   $x - y + 2z = 4$
   $x + y + z = 4$

2. $x + 2y + z = 7$
   $x - y + z = 4$
   $3x + 6y + 3z = 9$

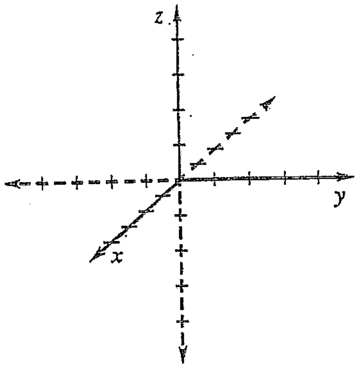
3. $x + y + z = 2$
   $x + y - z = 2$
   $2x + 2y + z = 4$

Advanced Algebra

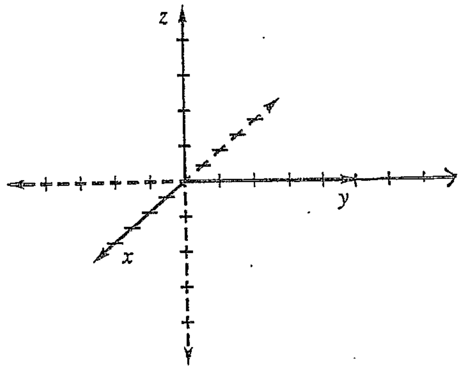
Name ___________________________

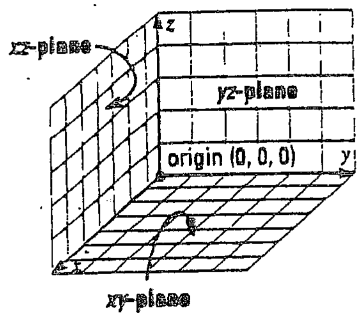
Plot the ordered triple in a three-dimensional coordinate system.

1) (2, 1, 4)

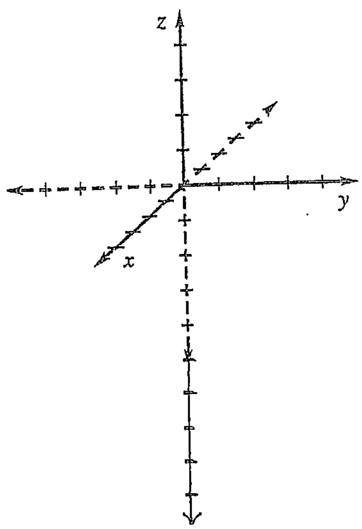


2) (-3, 4, -4)

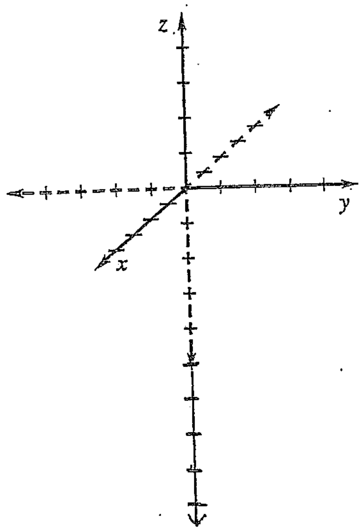




3) (3, -1, -5)

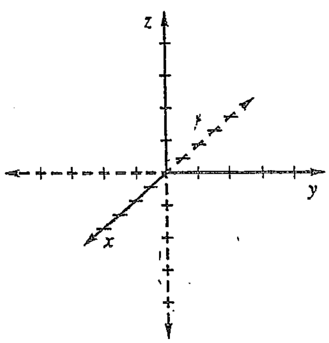


4) (3, 2, -4)



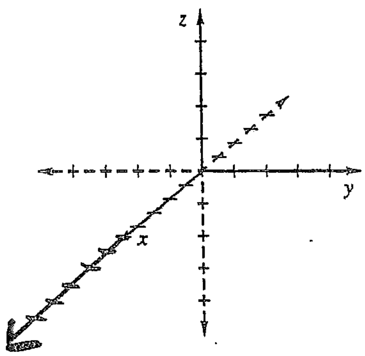
Sketch the graph of the equation. Label the three intercepts.

1. $x + y - z = 5$



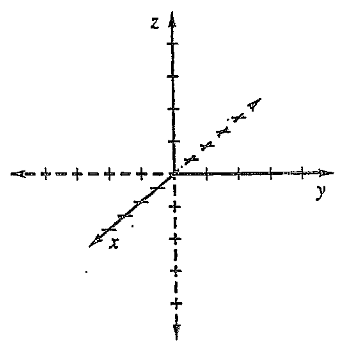
x	y	z
	0	0
0		0
0	0	

2. $x - 5y + 2z = 10$



x	y	z
	0	0
0		0
0	0	

3. $-2x + 4y - 8z = 8$



x	y	z
	0	0
0		0
0	0	

3-4

ELIMINATION METHOD **Solve the system using the elimination method.**

9. $3x + y + z = 14$
$-x + 2y - 3z = -9$
$5x - y + 5z = 30$

Section 3-4 Solve by elimination method.

1) $2x + y + z = 5$

$x - y + 2z = 4$

$x + y + z = 4$

2) $2x + 3y - z = 13$

$3x + y - 3z = 11$

$x - y + z = 3$

Advanced Algebra

Name_______________________

Date_____________ Hour____

Basic Matrix Functions 3.5

VOCABULARY:

A **matrix** is a rectangular arrangement of numbers in rows and columns. The **dimensions** of a matrix with m rows and n columns are m x n (read m by n). The numbers in a matrix are its **elements**.

Two matrices are equal if their dimensions are the same and the elements in corresponding positions are equal.

Adding and Subtracting Matrices:

To add or subtract two matrices, simply add or subtract elements in corresponding positions. You can add or subtract matrices on if they have the same dimensions.

Adding Matrices: $\begin{bmatrix} a & b \\ c & d \end{bmatrix} + \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} a+e & b+f \\ c+g & d+h \end{bmatrix}$

Subtracting Matrices: $\begin{bmatrix} a & b \\ c & d \end{bmatrix} - \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} a-e & b-f \\ c-g & d-h \end{bmatrix}$

Example:
Perform the indicated operation, if possible.

$$\begin{bmatrix} 3 & 0 \\ -5 & -1 \end{bmatrix} + \begin{bmatrix} -1 & 4 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} 3+(-1) & 0+4 \\ -5+2 & -1+0 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ -3 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 7 & 4 \\ 0 & -2 \\ -1 & 6 \end{bmatrix} - \begin{bmatrix} -2 & 5 \\ 3 & -10 \\ -3 & 1 \end{bmatrix} = \begin{bmatrix} 7-(-2) & 4-5 \\ 0-3 & -2-(-10) \\ -1-(-3) & 6-1 \end{bmatrix} = \begin{bmatrix} 9 & -1 \\ -3 & 8 \\ 2 & 5 \end{bmatrix}$$

Practice:
Perform the indicated operation, if possible.

1) $\begin{bmatrix} 4 & 6 \\ -2 & 2 \\ 1 & 5 \end{bmatrix} - \begin{bmatrix} 2 & 3 \\ -4 & 2 \\ -3 & 1 \end{bmatrix} =$

2) $\begin{bmatrix} 2 & 4 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} -3 & 2 \\ 4 & 0 \end{bmatrix} =$

3) $\begin{bmatrix} 1 & 7 \\ 4 & -3 \\ -3 & 1 \end{bmatrix} + \begin{bmatrix} 10 & -1 \\ -8 & 6 \end{bmatrix} =$

4) $\begin{bmatrix} 9 & 5 \\ -12 & 6 \\ 0 & -4 \end{bmatrix} + \begin{bmatrix} 3 & -5 \\ 7 & 1 \\ -3 & 2 \end{bmatrix} =$

Scalar Multiplication: In Matrix Algebra a real number is often called scalar. To multiply a matrix by a scalar, you multiply each element in the matrix by the scalar. This process is called scalar multiplication.

Example:
Perform the operation, if possible.

$$-2\begin{bmatrix} 4 & -1 \\ 1 & 0 \\ 2 & 7 \end{bmatrix} = \begin{bmatrix} -2(4) & -2(-1) \\ -2(1) & -2(0) \\ -2(2) & -2(7) \end{bmatrix} = \begin{bmatrix} -8 & 2 \\ -2 & 0 \\ -4 & -14 \end{bmatrix}$$

Practice:
Perform the operation, if possible.

5) $-4\begin{bmatrix} 2 & -3 \\ -7 & 1 \\ -2 & -5 \end{bmatrix} =$

6) $6\begin{bmatrix} -4 & 0 & -1 \\ 7 & 2 & 0 \\ -3 & 1 & 7 \end{bmatrix} + 3\begin{bmatrix} 9 & -2 & 5 \\ -1 & 4 & -3 \\ 10 & -5 & 1 \end{bmatrix} =$

Solving matrix equations: You can use what you already know about matrix operations and matrix equality to solve an equation involving matrices.

Example:

Solve the matrix equation for x and y.

$$3.\ 3\left(\begin{bmatrix} 5x & -2 \\ 6 & -4 \end{bmatrix} + \begin{bmatrix} 3 & 7 \\ -5 & -y \end{bmatrix}\right) = \begin{bmatrix} -21 & 15 \\ 3 & -24 \end{bmatrix}$$

Step 1: Write original equation

$$3\left(\begin{bmatrix} 5x & -2 \\ 6 & -4 \end{bmatrix} + \begin{bmatrix} 3 & 7 \\ -5 & -y \end{bmatrix}\right) = \begin{bmatrix} -21 & 15 \\ 3 & -24 \end{bmatrix}$$

Step 2: Add matrices inside parenthesis.

$$3\begin{bmatrix} 5x+3 & 5 \\ 1 & -4-y \end{bmatrix} = \begin{bmatrix} -21 & 15 \\ 3 & -24 \end{bmatrix}$$

ex

$$\begin{bmatrix} 3 & 5 \\ x & 2y \end{bmatrix} = \begin{bmatrix} 3 & 5 \\ 7 & 8 \end{bmatrix}$$

Step 3: Perform scalar multiplication.

$$\begin{bmatrix} 15x+9 & 15 \\ 3 & -12-3y \end{bmatrix} = \begin{bmatrix} -21 & 15 \\ 3 & -24 \end{bmatrix}$$

Equate corresponding elements and solve the two resulting equations.

$15x + 9 = -21; x = -2$ $\qquad$ $-12-3y = -24; y = 4$

The solution is $x = -2$ and $y = 4$

ex

$$\begin{bmatrix} 3 & 2 \\ x & y \end{bmatrix} + \begin{bmatrix} 3 & 5 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 6 & 7 \\ 5 & 8 \end{bmatrix}$$

Practice:

Solve the matrix for x and y.

$$7)\ 2\left(\begin{bmatrix} 2x & -3 \\ 5 & -y \end{bmatrix} + \begin{bmatrix} -1 & 4 \\ 3 & 5 \end{bmatrix}\right) = \begin{bmatrix} 10 & 2 \\ 16 & 14 \end{bmatrix}$$

Organizing data: Matrices are useful for organizing data and for performing the same operations on large numbers of data values.

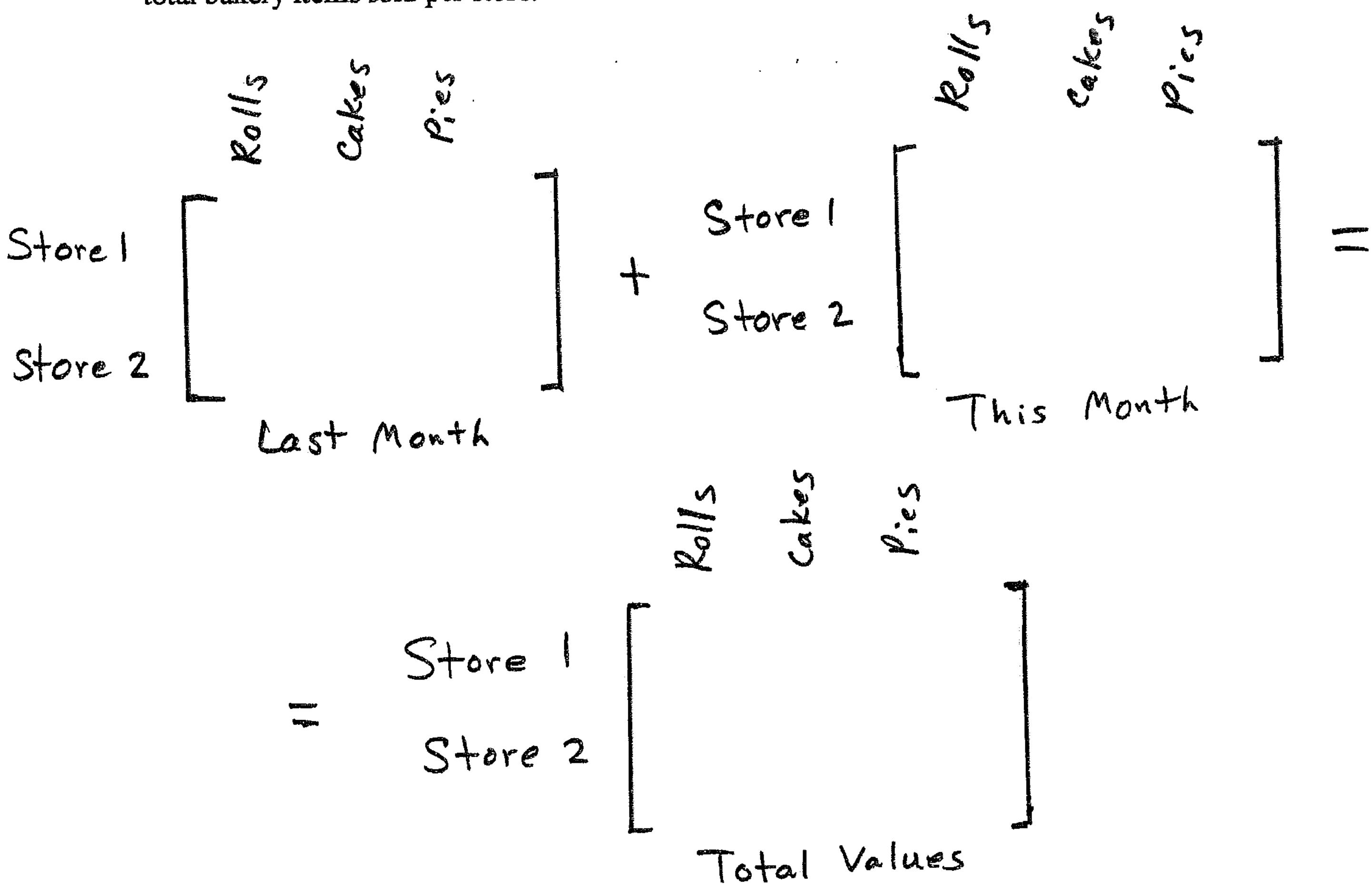
Practice/Example:

A local bakery keeps track of their sales as shown below.

Last Month Store 1: 650 Rolls, 220 Cakes, 32 pies
Store 2: 540 Rolls, 200 Cakes, 30 pies

This Month Store 1: 840 Rolls, 250 Cakes, 50 Pies
Store 2: 800 Rolls, 250 Cakes, 42 Pies

Organize the data using matrices. Then write and interpret a matrix giving the number of total bakery items sold per store.



Store 1 sold ______ rolls, ______ cakes, and ______ pies.

Store 2 sold ______ rolls, ______ cakes, and ______ pies.

Assignment 3-5 Page 191, #4, 9, 10, 16, 24, 26

Algebra 2 **Name** ______________________

Matrices Section 3-6 **Date** ____________ **Hour** ____

A · B = AB
× × ×
equal
dimensions of AB

GOAL **Multiply matrices.**

Find the product of two matrices

State whether the product *AB* is defined. If so, give the dimensions of *AB*.

a. A: 2×3, B: 4×3 **b.** A: 3×3, B: 3×2

Examples

State whether the product is defined. If so, give the dimensions.

1) A: 2 x 6 B: 6 x 5

2) A: 3 x 4 B: 3 x 4

3) A: 7 x 1 B: 1 x 1

4) A: 9 x 3 B: 3 x 2

5) A: 5 x 7 B: 5 x 7

6) A: 2 x 1 B: 1 x 2

7) A: 1 x 2 B: 2 x 1

Name ______________________________ Date ______________

LESSON 3.6

Study Guide

For use with pages 195–202

GOAL **Multiply matrices.**

EXAMPLE 1 Find the product of two matrices

Find AB if $A = \begin{bmatrix} 3 & 6 \\ 7 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 2 \\ -3 & 8 \end{bmatrix}$.

Because the number of columns in A (two) equals the number of rows in B (two), the product AB is defined and is a 2×2 matrix.

STEP 1 **Multiply** the numbers in the first row of A by the numbers in the first column of B, add the products, and put the result in the first row, first column of AB.

$$\begin{bmatrix} 3 & 6 \\ 7 & -1 \end{bmatrix}\begin{bmatrix} 4 & 2 \\ -3 & 8 \end{bmatrix} = \begin{bmatrix} 3(4) + 6(-3) & \\ & \end{bmatrix}$$

STEP 2 **Multiply** the numbers in the first row of A by the numbers in the second column of B, add the products, and put the result in the first row, second column of AB.

$$\begin{bmatrix} 3 & 6 \\ 7 & -1 \end{bmatrix}\begin{bmatrix} 4 & 2 \\ -3 & 8 \end{bmatrix} = \begin{bmatrix} 3(4) + 6(-3) & 3(2) + 6(8) \\ & \end{bmatrix}$$

STEP 3 **Multiply** the numbers in the second row of A by the numbers in the first column of B, add the products, and put the result in the second row, first column of AB.

$$\begin{bmatrix} 3 & 6 \\ 7 & -1 \end{bmatrix}\begin{bmatrix} 4 & 2 \\ -3 & 8 \end{bmatrix} = \begin{bmatrix} 3(4) + 6(-3) & 3(2) + 6(8) \\ 7(4) + (-1)(-3) & \end{bmatrix}$$

STEP 4 **Multiply** the numbers in the second row of A by the numbers in the second column of B, add the products, and put the result in the second row, second column of AB.

$$\begin{bmatrix} 3 & 6 \\ 7 & -1 \end{bmatrix}\begin{bmatrix} 4 & 2 \\ -3 & 8 \end{bmatrix} = \begin{bmatrix} 3(4) + 6(-3) & 3(2) + 6(8) \\ 7(4) + (-1)(-3) & 7(2) + (-1)(8) \end{bmatrix}$$

STEP 5 **Simplify** the product matrix.

$$\begin{bmatrix} 3(4) + 6(-3) & 3(2) + 6(8) \\ 7(4) + (-1)(-3) & 7(2) + (-1)(8) \end{bmatrix} = \begin{bmatrix} & \\ & \end{bmatrix}$$

Algebra 2 **Name** ______________________

Matrix Multiplication 3-6 **Date** ______________ **Hour** ____

Find the product.

1) $\begin{bmatrix} 5 & 3 \\ -3 & 5 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$

2) $\begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 5 & 3 \\ -3 & 5 \end{bmatrix}$

3) $\begin{bmatrix} 1 & 0 \\ -2 & -1 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 1 & 3 \end{bmatrix}$

4) $\begin{bmatrix} 1 & -4 \\ 3 & -2 \end{bmatrix} \begin{bmatrix} 4 & -1 \\ 0 & -3 \end{bmatrix}$

5) $\begin{bmatrix} -6 & -2 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} -1 & 4 \\ -5 & 3 \end{bmatrix}$

Find the product. If it is not defined, state the reason.

1. $\begin{bmatrix} 4 \\ -2 \end{bmatrix}\begin{bmatrix} 1 & 3 \end{bmatrix}$

2. $\begin{bmatrix} 2 & 3 \\ 4 & 7 \end{bmatrix}\begin{bmatrix} -5 & -2 \\ 1 & 6 \end{bmatrix}$

3. $\begin{bmatrix} 1 & -3 \\ 5 & 8 \end{bmatrix}\begin{bmatrix} -9 & 7 \\ -2 & -4 \end{bmatrix}$

PROPERTIES OF MATRIX MULTIPLICATION

Let A, B, and C be matrices and let k be a scalar.

Associative Property of Matrix Multiplication	$A(BC) =$ ________
Left Distributive Property	$A(B + C) =$ ___________
Right Distributive Property	$(A + B)C =$ ___________
Associative Property of Scalar Multiplication	$k(AB) =$ ________

Using the given matrices, evaluate the expression.

$A = \begin{bmatrix} -2 & 6 \\ 1 & 3 \end{bmatrix}, B = \begin{bmatrix} 0 & 2 \\ 4 & -5 \end{bmatrix}, C = \begin{bmatrix} 5 & -1 \\ -2 & 0 \end{bmatrix}$

4. $AB + C$

5. $2AB$

6. $A(B + C)$

7 *Use matrices to calculate total cost*

The school stores from the middle school and the high school each submit an inventory list for the year. Each sweatshirt costs $15, each T-shirt costs $9, and each pennant costs $5. Use matrix multiplication to find the total cost of the inventory for each school store.

Middle School: 61 sweatshirts, 63 T-shirts, and 74 pennants

High School: 58 sweatshirts, 71 T-shirts, and 92 pennants

Write the inventory and the cost in matrix form.

Inventory	Sweatshirt	T-shirt	Pennant
Middle	____	____	____
High	____	____	____

	Cost Dollars
Sweatshirt	____
T-shirt	____
Pennant	____

Find the total cost of inventory for each school store by multiplying the inventory matrix by the cost matrix.

$$\begin{bmatrix} 61 & 63 & 74 \\ 58 & 71 & 92 \end{bmatrix} \begin{bmatrix} 15 \\ 9 \\ 5 \end{bmatrix} = \underline{\qquad\qquad\qquad}$$

$$= \underline{\qquad}$$

Label the product matrix:

	Total Cost Dollars
Middle School	
High School	____

The total cost for the Middle School store is $______, and the total cost for the High School store is $______.

In Example 7, suppose that a sweatshirt costs $17, a T-shirt costs $11, and a pennant costs $7. Calculate the total cost of inventory for each school store.

$$\begin{bmatrix} \quad & \quad & \quad \\ \quad & \quad & \quad \end{bmatrix} \begin{bmatrix} \quad \\ \quad \\ \quad \end{bmatrix} = \begin{bmatrix} \quad \\ \quad \end{bmatrix}$$

M.S.

H.S.

Name ______________________ Date ______________

LESSON 3.6

Practice A

For use with pages 195–202

State whether the product AB is defined. If so, give the dimensions of AB.

1. $A: 2 \times 2, B: 2 \times 3$

2. $A: 4 \times 2, B: 2 \times 1$

3. $A: 3 \times 3, B: 2 \times 3$

4. $A: 1 \times 4, B: 3 \times 4$

5. $A: 5 \times 3, B: 3 \times 2$

6. $A: 1 \times 2, B: 2 \times 1$

Complete the remaining step(s) of the matrix multiplication.

7. $\begin{bmatrix} 3 & -1 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} 2 \\ 2 \end{bmatrix} = \begin{bmatrix} (3)(2) + (-1)(2) \\ ? \end{bmatrix}$

8. $\begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}\begin{bmatrix} 4 & 5 \end{bmatrix} = \begin{bmatrix} (3)(4) & (3)(5) \\ (2)(4) & ? \\ ? & ? \end{bmatrix}$

Find the product. If it is not defined, state the reason.

9. $\begin{bmatrix} 2 & 1 \end{bmatrix}\begin{bmatrix} 3 \\ 4 \end{bmatrix}$

10. $\begin{bmatrix} -2 & 0 \end{bmatrix}\begin{bmatrix} 4 & 1 \\ 2 & -1 \end{bmatrix}$

11. $\begin{bmatrix} -1 & 3 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix}$

12. $\begin{bmatrix} 4 \\ 2 \end{bmatrix}\begin{bmatrix} -2 & -4 \end{bmatrix}$

13. $\begin{bmatrix} 2 \\ 3 \end{bmatrix}\begin{bmatrix} -1 & 2 \\ -5 & 0 \end{bmatrix}$

14. $\begin{bmatrix} 2 & 1 \\ 5 & 2 \end{bmatrix}\begin{bmatrix} -2 & 3 \\ 0 & 2 \end{bmatrix}$

15. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 3 & -4 \\ -1 & 2 \end{bmatrix}$

16. $\begin{bmatrix} 1 & -3 & 4 \end{bmatrix}\begin{bmatrix} 4 \\ -1 \\ 2 \end{bmatrix}$

17. $\begin{bmatrix} 6 & -4 \\ 2 & 5 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

18. $\begin{bmatrix} 2 & 4 & -1 \\ 0 & 3 & 1 \end{bmatrix}\begin{bmatrix} 3 & 0 \\ -1 & 1 \\ 0 & 4 \end{bmatrix}$

19. $\begin{bmatrix} 2 & 1 & 1 \\ -2 & 3 & -1 \\ 0 & 2 & 4 \end{bmatrix}\begin{bmatrix} -3 & 1 & 0 \\ 2 & 2 & -4 \\ 1 & 0 & 1 \end{bmatrix}$

20. Football Attendance for the first three football games of the season is described in the table. Adult tickets sold for \$5.00. Student tickets sold for \$2.50. Use matrix multiplication to find the revenue for each game.

	Adults	Students
Game 1	320	150
Game 2	290	175
Game 3	350	220

Name ______________________ Date ______________

LESSON 3.6

Practice B

For use with pages 195–202

State the dimensions of each matrix and determine whether the product *AB* is defined. If so, give the dimensions of *AB*.

1. $A = \begin{bmatrix} 2 & 1 \\ 5 & 0 \\ 1 & 2 \end{bmatrix}, B = \begin{bmatrix} 2 & 1 & 5 \end{bmatrix}$

2. $A = \begin{bmatrix} 2 & -3 & 4 \\ -2 & 1 & 0 \end{bmatrix}, B = \begin{bmatrix} 1 & 2 & 0 \\ 5 & 4 & 3 \\ -4 & 2 & -5 \end{bmatrix}$

Find the product. If it is not defined, state the reason.

3. $\begin{bmatrix} 3 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 4 \end{bmatrix}$

4. $\begin{bmatrix} 1 & -1 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 2 & -2 \\ 1 & 2 \end{bmatrix}$

5. $\begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}\begin{bmatrix} -1 & 3 \end{bmatrix}$

6. $\begin{bmatrix} 3 & 5 \\ -2 & 4 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ 4 & 0 \end{bmatrix}$

7. $\begin{bmatrix} 5 & 1 & 0 \\ -2 & 3 & 1 \\ 0 & 2 & 4 \end{bmatrix}\begin{bmatrix} -1 & 2 & -3 \\ 0 & 5 & 4 \\ 2 & -1 & 2 \end{bmatrix}$

8. $\begin{bmatrix} 2 \\ 1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 4 \\ -2 & 3 & 1 \end{bmatrix}$

9. $\begin{bmatrix} -1 & 6 & 2 & 4 \end{bmatrix}\begin{bmatrix} 1 \\ -1 \\ 4 \\ 0 \end{bmatrix}$

10. $\begin{bmatrix} 2 & -3 \\ 4 & -2 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 2 & 1 & -1 \\ 5 & 3 & 2 \end{bmatrix}$

11. $\begin{bmatrix} 0 & 1 & 0 \\ 2 & 3 & -1 \\ -2 & 4 & 0 \\ 5 & 0 & -2 \end{bmatrix}\begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix}$

Using the given matrices, evaluate the expression.

$A = \begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 \\ 3 & -2 \end{bmatrix}, C = \begin{bmatrix} 3 & -2 \\ -1 & 4 \end{bmatrix}$

12. $-2BC$

13. $AC - AB$

14. $BA + BC$

15. **Football** Tickets to the football game cost $2.50 for students, $5.00 for adults, and $4.00 for senior citizens. Attendance for the first game of the postseason was 120 students, 185 adults, and 34 senior citizens. Attendance for the second game of the postseason was 150 students, 210 adults, and 50 senior citizens. Use matrix multiplication to find the revenue from ticket sales for each game.

Students Adults Senior Citizens

Game 1

Game 2

Algebra 2

Name____________________
Date______________ Hr_______

Scalar

Worksheet 3.5, Adding, Subtracting, and Multiplying Matrices

Perform the indicated operation, if possible. If not possible, state the reason.

1. $\begin{bmatrix} 2 & 1 \\ 6 & 4 \end{bmatrix} - \begin{bmatrix} -2 & 1 \\ -4 & 0 \end{bmatrix}$

2. $\begin{bmatrix} 5 & 2 \\ -1 & 4 \\ -3 & 6 \end{bmatrix} + \begin{bmatrix} -2 & 4 \\ 6 & -2 \\ 7 & -5 \end{bmatrix}$

3. $\begin{bmatrix} 6 & 4 & 3 \\ 1 & -3 & 2 \\ 8 & 7 & 1 \end{bmatrix} - \begin{bmatrix} 4 & 5 & -4 \\ 5 & 1 & 0 \\ 6 & 4 & 7 \end{bmatrix}$

4. $\begin{bmatrix} -4 & 2 & 3 \end{bmatrix} + \begin{bmatrix} -2 \\ 0 \\ -1 \end{bmatrix}$

5. $\begin{bmatrix} 10 & -5 & 7 \\ 2 & -12 & 0 \\ 8 & -4 & 6 \end{bmatrix} + \begin{bmatrix} -7 & 14 & 6 \\ 0 & 12 & -4 \\ 2 & 7 & 3 \end{bmatrix}$

6. $\begin{bmatrix} 10 & -7 & 14 \\ -5 & -10 & 0 \\ 9 & -3 & -7 \end{bmatrix} - \begin{bmatrix} -1 & -3 & 8 \\ -12 & 0 & 6 \\ 10 & -5 & 5 \end{bmatrix}$

Perform the indicated operation.

7. $-3\begin{bmatrix} 4 & 2 \\ 3 & 2 \end{bmatrix}$

8. $-2\begin{bmatrix} 3 & 0 & -1 \\ 0.5 & -6 & 4 \\ 7 & -1.25 & 9 \end{bmatrix}$

9. $-4\begin{bmatrix} 4 & 1 \\ -5 & 0 \\ 1 & -3 \end{bmatrix}$

Notes 3-5 Adding, Subtracting, and Scalar Multiplication of Matrices

Solve the matrix equation for x and y.

10. $\begin{bmatrix} -2x & 6 \\ 3y & 9 \end{bmatrix} = \begin{bmatrix} -8 & 6 \\ -12 & 9 \end{bmatrix}$

11. $\begin{bmatrix} 4 & 5x \\ -2 & 5 \end{bmatrix} + \begin{bmatrix} -11 & 2 \\ 6 & 5 \end{bmatrix} = \begin{bmatrix} y & 12 \\ 4 & 10 \end{bmatrix}$

In Exercises 12–15, use the following information.

Book Prices The matrices below show the number of books sold and the average price (in dollars) for the years 2002, 2003, and 2004.

	2002 (A)		2003 (B)		2004 (C)	
	Sold	Price	Sold	Price	Sold	Price
Book A	125,000	52.00	110,000	55.50	90,000	47.50
Book B	85,000	83.50	95,000	85.50	100,000	89.00
Book C	190,000	45.60	210,000	56.25	225,000	75.25

12. You purchased book A in 2002, book C in 2003, and book B in 2004. How much did you spend on these three books?

13. How many more (or less) volumes of book B were sold in 2004 than in 2002?

14. How much more (or less) is the price of book A in 2004 than in 2002?

15. In 2005, would you expect book C sales to be *more* or *less* than 100,000?

16. Solve.

$$4x - 5y + 2z = 2$$
$$-2x + 7y + 3z = 6$$
$$3x - 6y - z = -4$$

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3-4 Practice A Date __________________ Hour ___

Tell whether the given ordered triple is a solution of the system.

1. $(1, 1, 1)$

$2x - 3y + z = 0$

$2x - y + 2z = 3$

$x + y + z = 3$

2. $(1, 0, 2)$

$2x - y + z = 4$

$4x + 5y - z = 6$

$3x - y - 3z = -3$

Solve the system using the elimination method.

4. $x - 2y + 3z = 11$

$y - z = -3$

$2y + z = 0$

6. $3x + 3y + z = 1$

$2x - 3y + z = 2$

$x + y + z = 3$

8. $4x + 3y - 2z = -4$

$-4x + 2y - 5z = 10$

$x + 2y + z = 7$

3-4 Practice A

In Exercises 19 and 20, use the following information.

Movie Rates The movie theater charges different rates for attendees depending on their age; children 12 and under are $4, adults are $6 and senior citizens over 65 are $5. A group of 14 people from a family decides to go to the movies one weekend. There are an equal number of senior citizens as children 12 and under. The total cost was $66. Let x represent the number of children 12 and under. Let y represent the number of adults. Let z represent the number of senior citizens.

19. Write a system of linear equations in three variables to find the number of people in each age category in the group.

20. How many people in the group are in each age category?

Children:

Adults:

Seniors:

Algebra II

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Section 3.7

GOAL **Evaluate determinants of matrices.**

Vocabulary

Associated with each square matrix ($n \times n$) is a real number called its **determinant**. The determinant of a matrix A is denoted by det A.

Cramer's rule is a method for solving linear systems using determinants.

The **coefficient matrix** of the linear system $ax + by = e$ and $cx + dy = f$ is $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$.

THE DETERMINANT OF A MATRIX

Determinant of a 2 × 2 matrix

$$\det\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = \_\_\_\_ - \_\_\_\_$$

Determinant of a 3 × 3 matrix

$$\det\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} =$$

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}\begin{matrix} a & b \\ d & e \\ g & h \end{matrix} = (aei + bfg + cdh) - (gec + hfa + idb)$$

Evaluate the determinant of the matrix.

1. $\begin{bmatrix} -6 & 2 \\ 4 & -3 \end{bmatrix}$

2. $\begin{bmatrix} 7 & -2 \\ -4 & 3 \end{bmatrix}$

3. $\begin{bmatrix} 4 & 1 & 5 \\ -2 & 0 & 1 \\ 3 & 2 & -1 \end{bmatrix}$

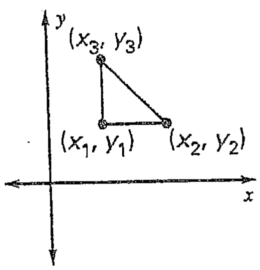
4. $\begin{bmatrix} -5 & 0 & 1 \\ 3 & -2 & 2 \\ 6 & 1 & 4 \end{bmatrix}$

(+ +) − (+ +)

AREA OF A TRIANGLE

The area of a triangle with vertices (x_1, y_1), (x_2, y_2), and (x_3, y_3) is given by

$$\text{Area} = \pm \;\underline{\quad\quad}\; \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$



where the symbol ± indicates that the appropriate sign should be chosen to yield a __________ value.

5. Find the area of the triangle with vertices $A(0, 0)$, $B(-5, 2)$, $C(3, 2)$.

CRAMER'S RULE FOR A 2 × 2 SYSTEM

Let A be the coefficient matrix of this linear system:

$ax + by = e$

$cx + dy = f$

If det $A \neq$ ____, then the system has ______________ solution.

$$x = \frac{\begin{vmatrix} e & b \\ f & d \end{vmatrix}}{\det A} \quad \text{and} \quad y = \frac{\begin{vmatrix} a & e \\ c & f \end{vmatrix}}{\det A}$$

Use Cramer's Rule to solve this system:

$3x + 2y = -4$

$2x - 7y = -11$

Evaluate the determinant of the _____________ matrix.

$$\begin{vmatrix} \_\_ & \_\_ \\ \_\_ & \_\_ \end{vmatrix} = \_\_\_\_\_\_\_\_\_\_\_\_$$

Apply Cramer's rule because the determinant is not ____.

$$x = \frac{\begin{vmatrix} \_\_ & \_\_ \\ \_\_ & \_\_ \end{vmatrix}}{\_\_\_\_} = \frac{\quad}{\_\_\_\_\_\_} = \_\_\_\_$$

$$y = \frac{\begin{vmatrix} \_\_ & \_\_ \\ \_\_ & \_\_ \end{vmatrix}}{\_\_\_\_} = \frac{\quad}{\_\_\_\_\_\_} = \_\_\_$$

The solution is (_____, ___).

Use Cramer's rule to solve the linear system.

6. $2x + y = 11$
$-3x + 2y = 1$

LESSON 3.7

Practice B

For use with pages 203–209

Evaluate the determinant of the matrix.

1. $\begin{bmatrix} 5 & -2 \\ 4 & -4 \end{bmatrix}$

2. $\begin{bmatrix} 3 & -5 \\ -2 & -3 \end{bmatrix}$

3. $\begin{bmatrix} \frac{1}{2} & \frac{2}{3} \\ 6 & 5 \end{bmatrix}$

Evaluate the determinant of the matrix. (use a calculator)

4. $\begin{bmatrix} 5 & -1 & 3 \\ 4 & 0 & 2 \\ 1 & -2 & -5 \end{bmatrix}$

5. $\begin{bmatrix} 3 & 2 & 9 \\ 0 & 1 & -4 \\ 5 & -1 & 2 \end{bmatrix}$

6. $\begin{bmatrix} 1 & 15 & 2 \\ 0 & 1 & 3 \\ 2 & 12 & 2 \end{bmatrix}$

7) **Gasoline** You fill up your car with 15 gallons of premium gasoline and fill up a 5 gallon gas can with regular gasoline for various appliances around the house. You pay the cashier \$42. The price of regular gasoline y is 20 cents less per gallon than the price of premium gasoline x.

a) Write a system of linear equations that models the price per gallon for regular and premium gasoline.

b) Use Cramer's rule to find the price per gallon of regular and premium gasoline.

Algebra II 3-7, Pages 207-208, #4-16 even, 22, 29, 30, 31, [redacted], (42 write system only)

CRAMER'S RULE FOR A 3 × 3 SYSTEM

Let A be the coefficient matrix of the linear system:

$ax + by + cz = j$

$dx + ey + fz = k$

$gx + hy + iz = \ell$

If $\det A \neq$ ___, then the system has ____________ solution.

$$x = \frac{\begin{vmatrix} j & b & c \\ k & e & f \\ \ell & h & i \end{vmatrix}}{\det A}, \quad y = \frac{\begin{vmatrix} a & j & c \\ d & k & f \\ g & \ell & i \end{vmatrix}}{\det A}, \quad z = \frac{\begin{vmatrix} a & b & j \\ d & e & k \\ g & h & \ell \end{vmatrix}}{\det A}$$

Use Cramer's rule to solve this system:

$3x + 4y - z = 9$

$-2x - 3y + 4z = -14$

$4x - y + z = -18$

Evaluate the determinant of the ____________ matrix.

$$\begin{vmatrix} \_ & \_ & \_ \\ \_ & \_ & \_ \\ \_ & \_ & \_ \end{vmatrix} \quad (\quad + \quad + \quad) - (\quad + \quad + \quad)$$

= ____

Apply Cramer's rule because the determinant is not ___.

x = ______ = ____ = ____

y = ______ = ____ = ____

z = ______ = ____ = ____

Name ____________________

Use Cramer's rule for a 3 × 3 system   Show all work!

8. $-6x + 4y + z = 32$
$5x + 2y + 3z = 13$
$x - y + z = -5$

Advanced Algebra

Name ____________________

Date ______________ Hour ____

Section 3.8

GOAL **Solve linear systems using inverse matrices.**

Vocabulary

The $n \times n$ **identity matrix** is a matrix with 1's on the main diagonal and 0's elsewhere. If A is any $n \times n$ matrix and I is the $n \times n$ identity matrix, then $AI = A$ and $IA = A$.

Two $n \times n$ matrices A and B are **inverses** of each other if their product (in both orders) is the $n \times n$ identity matrix.

In the matrix equation $AX = B$, matrix A is the coefficient matrix, X is the **matrix of variables**, and B is the **matrix of constants.**

THE INVERSE OF A 2 × 2 MATRIX

The inverse of the matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is

$$A^{-1} = \frac{1}{\square}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \frac{1}{\square}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

provided __________ $\neq 0$.

Find the inverse of $A = \begin{bmatrix} 3 & -2 \\ 4 & -4 \end{bmatrix}$.

$$A^{-1} = \frac{1}{\square}\begin{bmatrix} \_\_\_\_ \end{bmatrix}$$

$$= \_\_\_\_ \begin{bmatrix} \_\_\_\_ \end{bmatrix} = \begin{bmatrix} \_\_\_\_ \end{bmatrix}$$

Solve a matrix equation

Solve the matrix equation $AX = B$ for the 2×2 matrix X.

$$\overbrace{\begin{bmatrix} 1 & 1 \\ 6 & 7 \end{bmatrix}}^{A} X = \overbrace{\begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}}^{B}$$

Begin by finding the inverse of A.

$$A^{-1} = \frac{1}{7-6}\begin{bmatrix} 7 & -1 \\ -6 & 1 \end{bmatrix} = \begin{bmatrix} 7 & -1 \\ -6 & 1 \end{bmatrix}$$

To solve the equation for X, multiply both sides of the equation by A^{-1} on the left.

$$\begin{bmatrix} 7 & -1 \\ -6 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 6 & 7 \end{bmatrix} X = \begin{bmatrix} 7 & -1 \\ -6 & 1 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix} \qquad A^{-1}AX = A^{-1}B$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} X = \begin{bmatrix} 13 & 17 \\ -11 & -14 \end{bmatrix} \qquad IX = A^{-1}B$$

$$X = \begin{bmatrix} 13 & 17 \\ -11 & -14 \end{bmatrix} \qquad X = A^{-1}B$$

Solve the matrix equation.

1. $\begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix} X = \begin{bmatrix} 6 & 4 \\ 2 & 8 \end{bmatrix}$

2. $\begin{bmatrix} 1 & 1 \\ 3 & 4 \end{bmatrix} X = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}$

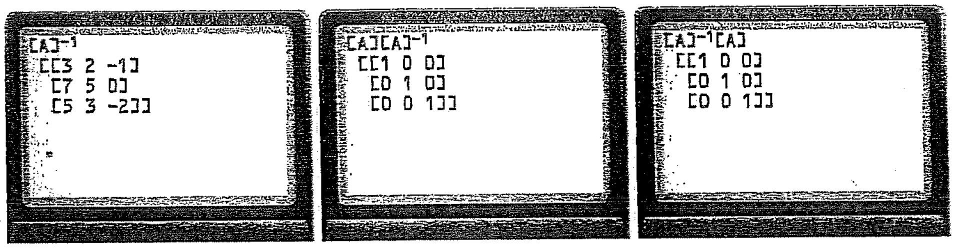
Find the inverse of a 3 × 3 matrix

Use a graphing calculator to find the inverse of *A*. Then use the calculator to verify your result.

$$A = \begin{bmatrix} 3 & 2 & -1 \\ 7 & 5 & 0 \\ 5 & 3 & -2 \end{bmatrix}$$

Solution

Enter the matrix *A* into a graphing calculator and calculate A^{-1}. Then compute _______ and _______ to verify that you obtain the _______ identity matrix.



1. Find the inverse of the matrix.

$$\begin{bmatrix} 6 & 4 \\ 4 & 3 \end{bmatrix}$$

2. Solve the matrix equation.

$$\begin{bmatrix} -7 & 3 \\ -5 & 2 \end{bmatrix} X = \begin{bmatrix} -6 & 9 \\ -7 & 5 \end{bmatrix}$$

3. Use a calculator to find the inverse of the matrix. Check the result.

$$\begin{bmatrix} -1 & 2 & 0 \\ 3 & -6 & 1 \\ -2 & 0 & 4 \end{bmatrix}$$

4. Use an inverse matrix to solve the linear system.

$$3x + 2y = 8$$
$$-2x - 5y = 2$$

Algebra II 3-8

Pages 214-215, #3, 4, 5, 13, 14 (w/o calc), 25, 28, 31, 35, 38 (with calculator)

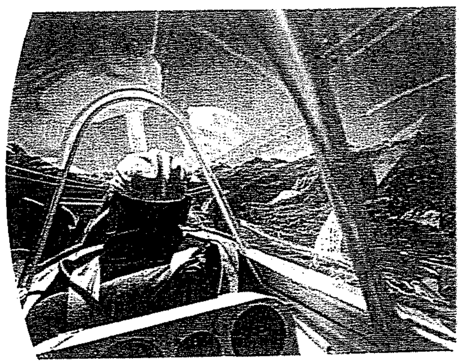
Use an inverse matrix to solve the linear systems. 3-8

You may use a calculator, but you must show what was entered into the calculator.

1) $6x + y = -2$
$-x + 3y = -25$

2) $2x + 4y + 5z = 5$
$x + 2y + 3z = 4$
$5x - 4y - 2z = -3$

3) **AVIATION** A pilot has 200 hours of flight time in single-engine airplanes and twin-engine airplanes. Renting a single-engine airplane costs \$60 per hour, and renting a twin-engine airplane costs \$240 per hour. The pilot has spent \$21,000 on airplane rentals. Use an inverse matrix to find how many hours the pilot has flown each type of airplane.



= hours in a single-engine plane

= hours in a twin-engine plane

The pilot spent hours flying a single-engine plane and hours flying a twin-engine plane.

Advanced Algebra

Name_________________________
Date________________Hr_________

Chapter 3 Review

1. Is (-3, 2) a solution to the system?

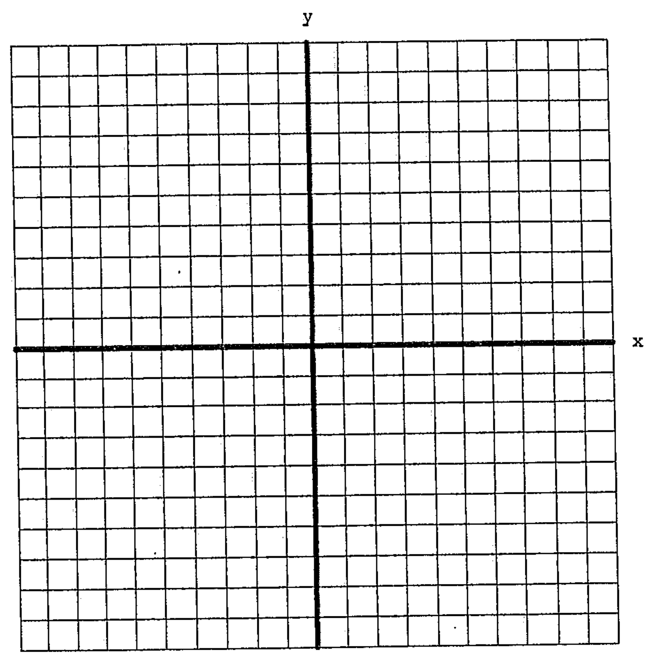
$$2x+4y=2$$
$$-x-3y=3$$

2. Is (1, -7) a solution to the system?

$$-2x+y=-9$$
$$4x-y=11$$

3. Graph the linear system and give the solution.

$$y=2x-7$$
$$y=-3x+3$$



Solve the following systems using an algebraic method.

4.
$$x=-2y+5$$
$$-2x+3y=-3$$

5.
$$2x-y=-4$$
$$-x+3y=-8$$

6. $\begin{aligned} 3x+2y&=-3 \\ -2x-3y&=7 \end{aligned}$

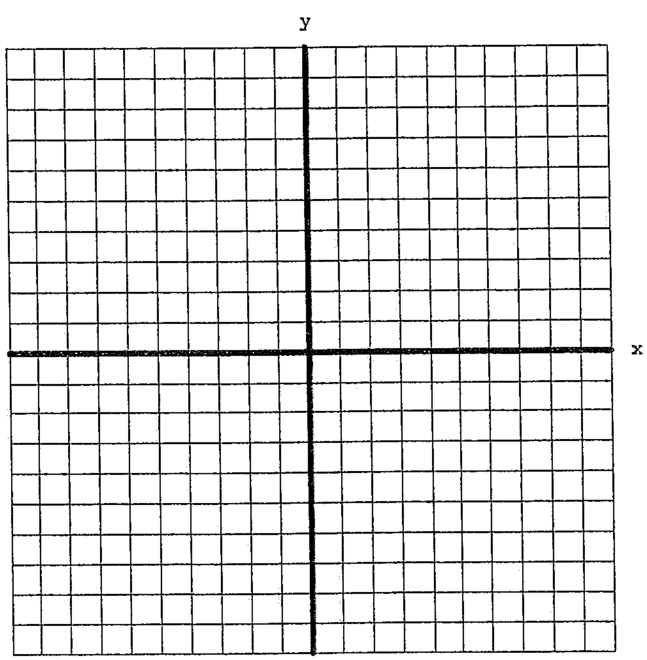
7. $\begin{aligned} -x-2y-z&=1 \\ 2x+3y+z&=0 \\ 3x+y+2z&=-1 \end{aligned}$

8. The student council is having a bake sale. They spend $40 dollars for the ingredients to make brownies and cookies. It costs $ 1.25 to make a dozen brownies and $1.50 to make a dozen cookies. The brownies sell for $6 per dozen and the cookies sell for $7.50 per dozen. The student council sells a total of $195 in baked goods. Let x be the number of dozens of brownies and y be the number of dozens of cookies. Set up a system of equations to represent the situation and then solve for x and y algebraically.

9. Graph the system of Linear inequalities.

$y > -2x$

$y < -2x+2$



10. Find the sum of the matrices.

$$\begin{bmatrix} 7 & 3 \\ 1 & -2 \end{bmatrix} + \begin{bmatrix} -9 & -4 \\ 1 & 11 \end{bmatrix} =$$

11. Find the difference of the matrices.

$$\begin{bmatrix} -1 & 7 & -1 \\ 7 & 9 & 5 \\ -2 & 10 & 4 \end{bmatrix} - \begin{bmatrix} 3 & 8 & 6 \\ 3 & -5 & 3 \\ 6 & 11 & -6 \end{bmatrix} =$$

Find the product.

12.

$$-4 \begin{bmatrix} 3 & 7 \\ -2 & -6 \end{bmatrix}$$

13. Show work!

$$\begin{bmatrix} 4 & 3 \\ 3 & -1 \end{bmatrix} \begin{bmatrix} 8 & 2 \\ -4 & -7 \end{bmatrix} =$$

14. A radio station has three types of promotional items: CDs, hats, and flying disks. CDs are worth $4.50, hats are worth $3.25, and flying disks are worth $1.25. They have given away 11 CDs, 26 hats, and 53 flying disks. Organize the information using matrices. Then use matrix multiplication to find the stations total cost for promotional items.

$$A^{-1} = \frac{1}{|A|}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

15. Find the determinant of the matrix.
No Calculator. Show all work.

$$\begin{bmatrix} -2 & -3 \\ 4 & -2 \end{bmatrix}$$

16. Find the inverse of the matrix.
No Calculator. Show all work.

$$\begin{bmatrix} -2 & 6 \\ -1 & 5 \end{bmatrix}$$

17. Use Cramer's rule to solve the system of equations. Show all substitutions into Cramer's rule.

$$2x + 6y = 12$$
$$-x + 7y = -6$$

Use an inverse matrix to solve the linear systems. You may use a calculator, but you must show what was entered into the calculator.

18. $4x + 5y = 7$
$-2x - y = 7$

19. $3x + y - 2z = 3$
$-2x + 2y + 3z = -14$
$5x - y - 3z = 25$