

4th

Algebra 2A

Chapter 4 Packet
(2nd half)

Name _____

Exponent Rules

1. Zero Exponent..... $b^0 = 1, b \neq 0$

2. Negative Exponent..... $b^{-n} = \frac{1}{b^n} \Leftrightarrow b^n = \frac{1}{b^{-n}}$

3. Product of Powers..... $b^m \cdot b^n = b^{m+n}$

4. Quotient of Powers..... $\frac{b^m}{b^n} = b^{m-n}$

5. Power of a Power..... $(b^m)^n = b^{m \cdot n}$

6. Power of a Product..... $(ab)^n = a^n b^n$

7. Power of a Quotient..... $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

NOTE: $(x \pm y)^3 \neq x^3 \pm y^3$

Algebra

Name _____

Date _____ Hour _____

Simplify the radicals.

1) $\sqrt{48}$

2) $\sqrt{8}$

3) $\sqrt{32}$

4) $\sqrt{20}$

5) $\sqrt{52}$

6) $\sqrt{12}$

7) $\sqrt{50}$

8) $\sqrt{128}$

NAME: _____

SIMPLIFYING RADICALS # 3

1.) $\sqrt{45}$

6.) $\sqrt{56}$

2.) $\sqrt{125}$

7.) $\sqrt{147}$

3.) $\sqrt{40}$

8.) $\sqrt{76}$

4.) $\sqrt{63}$

9.) $\sqrt{175}$

5.) $\sqrt{44}$

10.) $\sqrt{117}$

Properties of Exponents

Properties of Exponents:

Product of Powers: $a^m \cdot a^n = a^{m+n}$

Power of a Power: $(a^m)^n = a^{mn}$

Power of a Product: $(ab)^m = a^m b^m$

Negative Exponent: $a^{-m} = \frac{1}{a^m}$

Zero Exponent: $a^0 = 1, a \neq 0$

1. $4^2 \cdot 4^3 =$

2. $(6^2)^4 =$

3. $(-3 \cdot 2^5)^2 =$

4. $(2^2)^6 =$

5. $5^{-4} =$

6. $3^4 \cdot 3^2 =$

7. $16^0 =$

8. $(4^3 \cdot 3^2)^3 =$

9. $2^{-6} =$

10. $(-3^3)^5$

11. $3^0 =$

12. $2^3 \cdot 2^5 =$

Properties of Exponents

Properties of Exponents:

Quotient of Powers: $\frac{a^m}{a^n} = a^{m-n}, a \neq 0$

Power of a Quotient: $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}, b \neq 0$

Negative Exponent: $a^{-m} = \frac{1}{a^m}$

Zero Exponent: $a^0 = 1, a \neq 0$

1. $\frac{3^7}{3^4} =$

2. $8^{-2} =$

3. $\left(\frac{2}{3}\right)^4 =$

4. $-8^0 =$

5. $\frac{8^4}{8^2} =$

6. $\frac{11^{10}}{11^6} =$

7. $\left(\frac{5}{6}\right)^3 =$

8. $10^{-5} =$

9. $\left(\frac{9}{10}\right)^{-3} =$

10. $100^0 =$

11. $\frac{6^{12}}{6^4} =$

12. $\left(\frac{4}{5}\right)^{-5} =$

4.5, Solve Quadratic Equations by Finding Square Roots

GOAL Solve quadratic equations by finding square roots.

Vocabulary

A number r is a **square root** of a number s if $r^2 = s$.

The expression \sqrt{s} is called a **radical**. The symbol $\sqrt{\quad}$ is a radical sign and the number s beneath the radical sign is the **radicand** of the expression.

To **rationalize a denominator** \sqrt{b} of a fraction, multiply the numerator and denominator by \sqrt{b} . To **rationalize a denominator** $a + \sqrt{b}$ of a fraction, multiply the numerator and denominator by $a - \sqrt{b}$, and to **rationalize a denominator** $a - \sqrt{b}$ of a fraction, multiply the numerator and denominator by $a + \sqrt{b}$.

The expressions $a + \sqrt{b}$ and $a - \sqrt{b}$ are called **conjugates**.

PROPERTIES OF SQUARE ROOTS ($a > 0, b > 0$)

Example

Product Property $\sqrt{ab} = \underline{\quad} \cdot \underline{\quad}$ $\sqrt{18} = \sqrt{9} \cdot \sqrt{2} = 3\sqrt{2}$

Quotient Property $\sqrt{\frac{a}{b}} = \frac{\boxed{\quad}}{\boxed{\quad}}$ $\sqrt{\frac{2}{25}} = \frac{\sqrt{2}}{\sqrt{25}} = \frac{\sqrt{2}}{5}$

Example 1 Use properties of square roots

a. $\sqrt{24} = \underline{\quad} \cdot \underline{\quad} = \underline{\quad}$

b. $\sqrt{5} \cdot \sqrt{18} = \underline{\quad} = \underline{\quad} \cdot \underline{\quad} = \underline{\quad}$

c. $\sqrt{\frac{9}{64}} = \frac{\boxed{\quad}}{\boxed{\quad}} = \underline{\quad}$

Example 3 Solve a quadratic equationSolve $\frac{1}{4}(y - 6)^2 = 8$.

$$\frac{1}{4}(y - 6)^2 = 8$$

Original equation

$$\underline{\hspace{2cm}} = \underline{\hspace{2cm}}$$

Multiply each side by $\underline{\hspace{1cm}}$.

$$\underline{\hspace{2cm}} = \underline{\hspace{2cm}}$$

Take square roots of each side.

$$y = \underline{\hspace{2cm}}$$

Add $\underline{\hspace{1cm}}$ to each side.

$$y = \underline{\hspace{2cm}}$$

Simplify.

The solutions are $\underline{\hspace{2cm}}$ and $\underline{\hspace{2cm}}$.

Solve the equation.

7. $-9d^2 = -405$

8. $11y^2 + 3 = 36$

9. $\frac{1}{2}(w - 2)^2 + 1 = 4$

Simplify.

10.
$$\frac{2}{4 + \sqrt{11}}$$

11.
$$\frac{4}{8 - \sqrt{3}}$$

Example 4 *Model a dropped object with a quadratic function*

Water Balloon A water balloon is dropped from a window 59 feet above the sidewalk. How long does it take for the water balloon to hit the sidewalk?

Solution

$$h = -16t^2 + h_0$$

Write height function.

$$\underline{\hspace{2cm}} = -16t^2 + \underline{\hspace{2cm}}$$

Substitute $\underline{\hspace{2cm}}$ for h and $\underline{\hspace{2cm}}$ for h_0 .

$$\underline{\hspace{2cm}} = -16t^2$$

Subtract $\underline{\hspace{2cm}}$ from each side.

$$\underline{\hspace{2cm}} = t^2$$

Divide each side by $\underline{\hspace{2cm}}$.

$$\underline{\hspace{2cm}} = t$$

Take square roots of each side.

$$\underline{\hspace{2cm}} \approx t$$

Use a calculator. *round nearest tenth*

Reject the negative solution, $\underline{\hspace{2cm}}$, because time must be positive. The water balloon will fall for about $\underline{\hspace{2cm}}$ seconds before it hits the ground.

In Example 4, suppose that the water balloon is dropped from a height of 27 feet. How long does it take for the balloon to hit the sidewalk?

round nearest tenth

4.6, Perform Operations with Complex Numbers

GOAL Perform operations with complex numbers.

Vocabulary

The **imaginary unit** i is defined as $i = \sqrt{-1}$.

A **complex number** written in standard form is a number $a + bi$ where a and b are real numbers. If $b \neq 0$, then $a + bi$ is an **imaginary number**.

Two complex numbers of the form $a + bi$ and $a - bi$ are called **complex conjugates**.

Every complex number corresponds to a point in the **complex plane**. The complex plane has a horizontal axis called the real axis and a vertical axis called the imaginary axis.

The **absolute value** of a complex number $z = a + bi$, denoted $|z|$, is a nonnegative real number defined as $|z| = \sqrt{a^2 + b^2}$.

THE SQUARE ROOT OF A NEGATIVE NUMBER

Property

Example

1. If r is a positive real number, then $\sqrt{-r} = i\sqrt{r}$.

$$\sqrt{-3} = \underline{\hspace{2cm}}$$

2. By Property (1), it follows that $(i\sqrt{r})^2 = -r$.

$$(i\sqrt{3})^2 = i^2 \cdot 3 \\ = \underline{\hspace{2cm}}$$

SUMS AND DIFFERENCES OF COMPLEX NUMBERS

To add (or subtract) two complex numbers, add (or subtract) their _____ parts and their _____ parts _____.

Sum of complex numbers:

$$(a + bi) + (c + di) = (a + c) + (b + d)i$$

Difference of complex numbers:

$$(a + bi) - (c + di) = (a - c) + (b - d)i$$

Solve the equation.

1. $7x^2 - 13 = -20$

2. $x^2 + 14 = 2$

3. $4x^2 - 5 = -77$

Write the expression as a complex number in standard form.

4. $(-11 + 3i) + (4 - 6i)$

5. $15 - (9 + 4i) - 7i$

Example 3 *Multiply complex numbers*

Write the expression $(2 + i)(-5 + 2i)$ as a complex number in standard form.

$(2 + i)(-5 + 2i)$

$=$ _____

Multiply using FOIL.

$=$ _____

Simplify and use $i^2 = -1$.

$=$ _____

Simplify.

$=$ _____

Write in standard form.

Example 4**Divide complex numbers**

Write the quotient $\frac{6 + 4i}{2 + i}$ in standard form.

$$\frac{6 + 4i}{2 + i} = \frac{6 + 4i}{2 + i} \cdot \underline{\hspace{2cm}}$$

Multiply numerator and denominator by $\underline{\hspace{2cm}}$, the complex conjugate of $2 + i$.

$$= \underline{\hspace{4cm}}$$

Multiply using FOIL.

$$= \underline{\hspace{4cm}}$$

Simplify.

$$= \underline{\hspace{4cm}}$$

Write in standard form.

Write the product or quotient in standard form.

6. $-8i(3 - i)$

7. $(-3 + 5i)(4 - 2i)$

8. $(7 + 6i)(7 - 6i)$

9. $\frac{1 + 3i}{2i}$

10. $\frac{2i}{3 + i}$

11. $\frac{4 + 2i}{1 - i}$

Example 5 Plot complex numbers

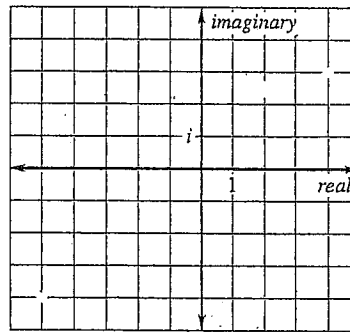
Plot the complex numbers in the same complex plane.

- a. $4 + 3i$ b. $-5 - 4i$

Solution

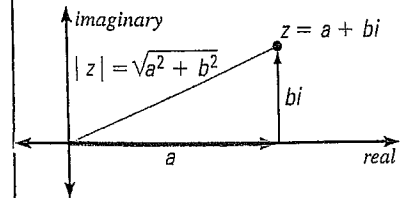
a. To plot $4 + 3i$, start at the origin, move _____, and then move _____.

b. To plot $-5 - 4i$, start at the origin, move _____, and then move _____.



ABSOLUTE VALUE OF A COMPLEX NUMBER

The absolute value of a complex number $z = a + bi$, denoted $|z|$, is a _____ real number defined as $|z| = \sqrt{a^2 + b^2}$. This is the distance of z from the _____ in the complex plane.



Example 6 Find absolute values of complex numbers

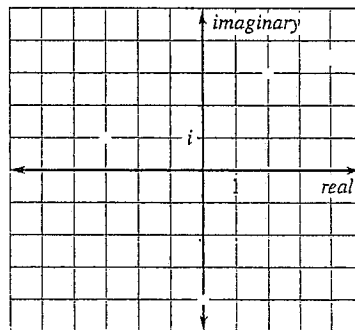
Find the absolute value of (a) $6 - 8i$ and (b) $-6i$.

a. $|6 - 8i| = \sqrt{\quad} = \sqrt{\quad} = \quad$

b. $|-6i| = \sqrt{\quad} = \sqrt{\quad} = \quad$




Plot the complex numbers in the same complex plane. Then find the absolute value.

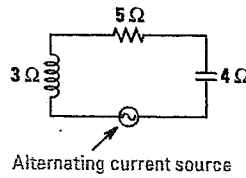
- $-4i$
-
- $2 + 3i$
-
- $-3 + i$



EXAMPLE 3 Use addition of complex numbers in real life

ELECTRICITY Circuit components such as resistors, inductors, and capacitors all oppose the flow of current. This opposition is called *resistance* for resistors and *reactance* for inductors and capacitors. A circuit's total opposition to current flow is *impedance*. All of these quantities are measured in ohms (Ω).

Component and symbol	Resistor 	Inductor 	Capacitor 
Resistance or reactance	R	L	C
Impedance	R	iL	$-iC$



The table shows the relationship between a component's resistance or reactance and its contribution to impedance. A *series circuit* is also shown with the resistance or reactance of each component labeled.

The impedance for a series circuit is the sum of the impedances for the individual components. Find the impedance of the circuit shown above.

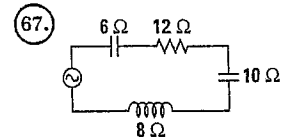
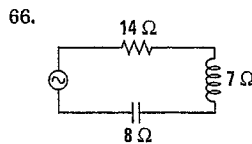
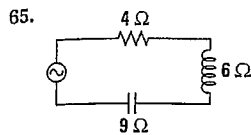
Resistor ohms

Inductor ohms

Capacitor ohms

Impedance of circuit
(add individual impedances)
simplify

CIRCUITS In Exercises 65–67, each component of the circuit has been labeled with its resistance or reactance. Find the impedance of the circuit.



Algebra II

Name _____

Equation Solving 4.3-4.6

Date _____ Hour _____

1) $x^2 + 2x - 35 = 0$

2) $x^2 + 10x + 25 = 0$

3) $x^2 + 2x = 80$

4) $-3x^2 + 28 = x^2$

5) $11x^2 - 19x - 6 = 0$

6) $2x^2 - x - 21 = 0$

$$7) \quad x^2 - 7 = 29$$

$$8) \quad 7(x-4)^2 - 18 = 10$$

$$9) \quad \frac{x^2}{25} - 6 = -2$$

$$10) \quad 6x^2 + 5 = 2x^2 + 1$$

$$11) \quad -5(x-3)^2 = 10$$

$$12) \quad 9 - 4x^2 = 57$$

Algebra II 4-6 Complex Numbers

1. For the complex number $2 + 7i$, identify the real part and the imaginary part.
2. Solve the equation. $4x^2 + 20 = 0$
3. Solve the equation. $4x^2 + 5 = -7$

Write the expression as a complex number in standard form.

4. $(-2 + 4i) - (3 + 9i)$
5. $(-3 - 8i) + (-5 - 7i)$
6. $(5 - 2i) - 2(3 + i)$
7. $(5 - 2i) + (3 - 2i)$
8. $-i + (7 - 5i) - 3(2 - 3i)$
9. $i(2 + i)$

Write the expression as a complex number in standard form.

ID: A

10. $3i(6-5i)$

11. $(2+3i)(1-4i)$

12. $(-3+7i)(1-2i)$

13. $(3-2i)^2$

14. $\frac{5}{1+i}$

15. $\frac{3-3i}{4i}$

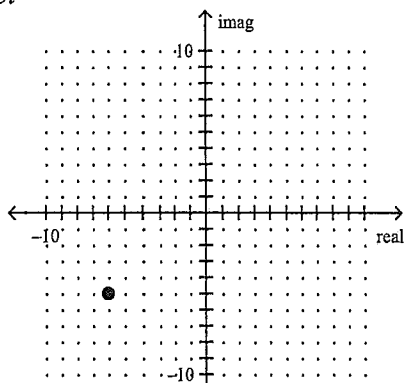
16. $\frac{-1+10i}{-9i}$

17. $\frac{8+7i}{3-4i}$

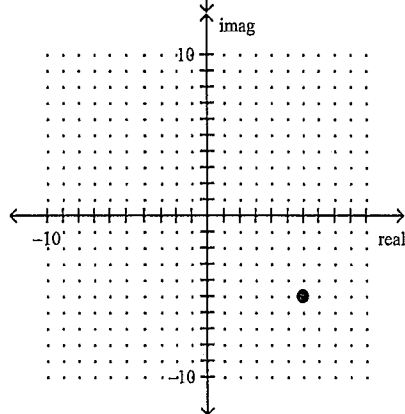
Plot the number in a complex plane.

_____ 18. $6 - 5i$

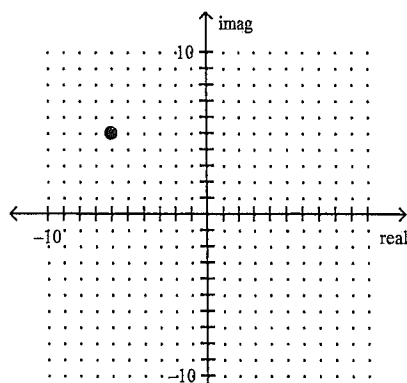
a.



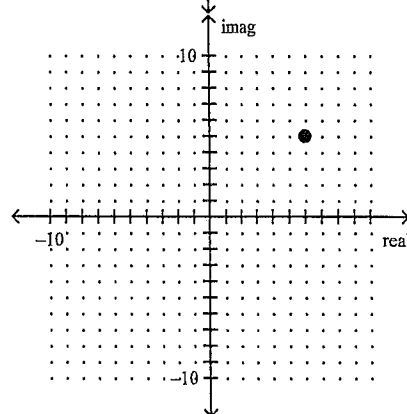
b.



c.



d.



19. Write the expression $(5 + 7i)(5 - 7i)$ as a complex number in standard form.

20. Write the expression $(1 - i)(1 + i)$ as a complex number in standard form.

21. Write the expression $(3 - 2i)(3 + 2i)$ as a complex number in standard form.

4.7, Complete the Square

GOAL Solve quadratic equations by completing the square.

Vocabulary
 To complete the square for the expression $x^2 + bx$, add $\left(\frac{b}{2}\right)^2$.
 When you complete the square, $x^2 + bx + \left(\frac{b}{2}\right)^2 = \left(x + \frac{b}{2}\right)^2$.

Example 1 *Make a perfect square trinomial*

Find the value of c that makes $x^2 + 12x + c$ a perfect square trinomial. Then write the expression as the square of a binomial.

Solution

1. Find half the coefficient of x . $\frac{\square}{2} = \underline{\hspace{2cm}}$
2. Square the result of Step 1. $\underline{\hspace{2cm}} = \underline{\hspace{2cm}}$
3. Replace c with the result of Step 2. $\underline{\hspace{2cm}}$

The trinomial $x^2 + 12x + c$ is a perfect square when $c = \underline{\hspace{2cm}}$. Then
 $\underline{\hspace{2cm}} = (\underline{\hspace{2cm}})(\underline{\hspace{2cm}}) = (\underline{\hspace{2cm}})^2$.

Find the value of c that makes the expression a perfect square trinomial. Then write the expression as the square of a binomial.

1. $x^2 + 10x + c$
2. $x^2 - 18x + c$
3. $x^2 - 40x + c$

Example 2 Solve $ax^2 + bx + c = 0$ when $a = 1$

Solve $x^2 - 10x + 13 = 0$ by completing the square.

Solution

$$x^2 - 10x + 13 = 0$$

Write original equation.

$$\underline{\hspace{2cm}} = \underline{\hspace{2cm}}$$

Write left side in the form $x^2 + bx$.

$$\underline{\hspace{2cm}} = \underline{\hspace{2cm}}$$

Complete the square.

$$\underline{\hspace{2cm}} = \underline{\hspace{2cm}}$$

Write left side as a binomial squared.

$$\underline{\hspace{2cm}} = \underline{\hspace{2cm}}$$

Take square roots of each side.

$$x = \underline{\hspace{2cm}}$$

Solve for x .

$$x = \underline{\hspace{2cm}}$$

Simplify.

The solutions are $\underline{\hspace{2cm}}$ and $\underline{\hspace{2cm}}$.

Solve the equation by completing the square.

4. $x^2 - 10x + 6 = 0$ 5. $2x^2 + 16x + 8 = 0$ 6. $5x^2 - 10x + 30 = 0$

Example 3**Write a quadratic function in vertex form**

Write $y = x^2 + 14x + 44$ in vertex form. Then identify the vertex.

Solution

$$y = x^2 + 14x + 44$$

Write original function.

$$y + \underline{\hspace{2cm}} = (x^2 + 14x + \underline{\hspace{2cm}}) + 44$$

Complete the square.

$$y + \underline{\hspace{2cm}} = (\underline{\hspace{2cm}})^2 + 44$$

Write as a binomial squared.

$$y = \underline{\hspace{4cm}}$$

Solve for y.

The vertex form of the function is $y = \underline{\hspace{4cm}}$.

The vertex is $(\underline{\hspace{1cm}}, \underline{\hspace{1cm}})$.

Write the equation in vertex form and identify the vertex.

7. $y = x^2 - 12x + 38$

8. $y = x^2 - 14x + 50$

9. $y = 2x^2 + 12x + 13$

Example 7 (page 287)

The height y (in feet) of a baseball t seconds after it is hit is given by this function:

$$y = -16t^2 + 96t + 3$$

Find the maximum height of the baseball.

(The maximum height is the y -coordinate of the vertex of the parabola.)

$$y = -16t^2 + 96t + 3$$

- 62) While marching, a drum major tosses a baton into the air and catches it. The height h (in feet) of the baton after t seconds can be modeled by $h = -16t^2 + 32t + 6$. Find the maximum height of the baton.

$$h = -16t^2 + 32t + 6$$

Name _____

Date _____

LESSON
4.5**Practice B**

For use with pages 266–271

Simplify the expression.

1. $\sqrt{242}$

2. $\sqrt{153}$

3. $\sqrt{56}$

4. $5\sqrt{24} \cdot 2\sqrt{28}$

5. $\sqrt{8} \cdot 3\sqrt{40} \cdot \sqrt{3}$

6. $\sqrt{10} \cdot \sqrt{14}$

7. $\sqrt{\frac{121}{225}}$

8. $\sqrt{\frac{7}{9}} \cdot \sqrt{\frac{4}{7}}$

9. $\sqrt{24} \cdot \sqrt{\frac{80}{192}}$

10. $\frac{3}{4 + \sqrt{5}}$

11. $\frac{-6}{5 - \sqrt{11}}$

12. $\frac{7 - \sqrt{7}}{10 + \sqrt{3}}$

Solve the equation.

13. $x^2 = 289$

14. $x^2 - 169 = 0$

15. $2x^2 - 512 = 0$

16. $3x^2 - 150 = 282$

17. $\frac{1}{2}x^2 - 8 = 16$

18. $\frac{2}{3}x^2 - 4 = 12$

Solve the equation.

19. $2x^2 + 5 = 5x^2 - 37$

20. $4(x^2 - 8) = 84$

21. $3(x^2 + 2) = 18$

22. $2(x + 2)^2 = 72$

23. $3(x - 3)^2 + 2 = 26$

24. $(3x + 2)^2 - 49 = 0$

25. $(4x - 5)^2 = 64$

26. $\frac{1}{2}(x - 4)^2 = 8$

27. $\frac{2}{3}(x + 8)^2 - 66 = 0$

When an object is dropped, its height h can be determined after t seconds by using the falling object model $h = -16t^2 + s$ where s is the initial height. Find the time it takes an object to hit the ground when it is dropped from a height of s feet.

round to
nearest
hundredth
(2 decimal
places)

28. $s = 160$

29. $s = 300$

30. $s = 550$

31. $s = 690$

32. $s = 900$

33. $s = 1600$

Name _____

Date _____

LESSON
4.6 **Practice B**
For use with pages 275–282

Solve the equation.

1. $x^2 = -36$

2. $x^2 + 121 = 0$

3. $x^2 + 9 = 4$

4. $x^2 = 2x^2 + 4$

5. $3x^2 + 40 = -x^2 - 56$

6. $11x^2 = -5x^2 - 1$

7. $(x - 3)^2 = -12$

8. $-2(x - 1)^2 = 36$

9. $4(x + 2)^2 + 320 = 0$

Write the expression as a complex number in standard form.

 $a + bi$

10. $(1 + i) + (3 + i)$

11. $(4 - 3i) + (2 + 6i)$

12. $(-4 - i) - (4 + 5i)$

13. $(5 - 3i) + (-3 - 6i)$

14. $3i(4 + 2i)$

15. $-2i(3 - i)$

Write the expression as a complex number in standard form. $a + bi$

16. $(2 + i)(4 + 2i)$

17. $(5 - 2i)(1 - 3i)$

18. $-(3 + i)(7 - 3i)$

19. $-2i(1 + i)(2 + 3i)$

20. $(2 - i)^2$

21. $(5 + 3i)(5 - 3i)$

22. $\frac{5}{3 - 2i}$

23. $\frac{2 - i}{3 + 4i}$

24. $\frac{1 + 2i}{\sqrt{2} + i}$

25. $\frac{3}{2 - 4i} - (3 + 2i)$

Find the absolute value of the complex number.

$$\sqrt{a^2 + b^2}$$

26. $3 - 4i$

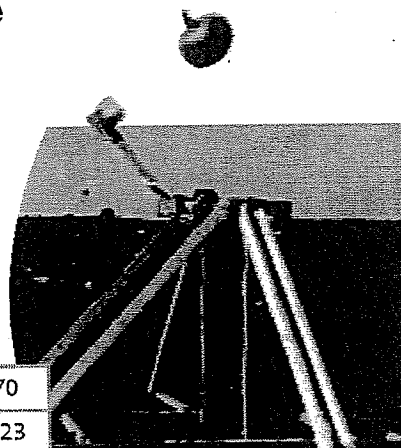
27. $1 - i\sqrt{3}$

28. $\sqrt{5} + 2i\sqrt{2}$

4-10 Quadratic Regression

Pumpkin Tossing

A pumpkin tossing contest is held each year in Morton, Illinois, where people compete to see whose catapult will send pumpkins the farthest. One catapult launches pumpkins from 25 feet above the ground at a speed of 125 feet per second.



Angle (degrees)	20	30	40	50	60	70
Distance (feet)	372	462	509	501	437	323

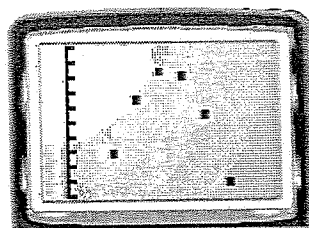
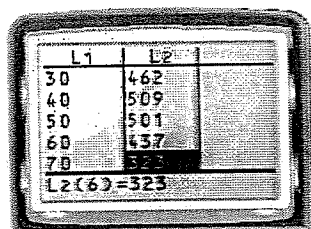
The table shows the horizontal distances (in feet) the pumpkins travel when launched at different angles. Use a graphing calculator to find the best-fitting quadratic model for the data.

SOLUTION**STEP 1**

Enter the data into two lists of a graphing calculator.

STEP 2

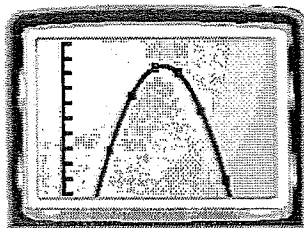
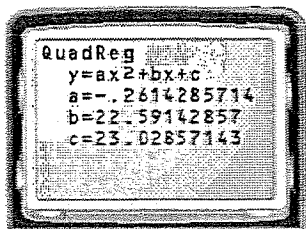
Make a scatter plot of the data. Note that the points show a parabolic trend.



STEP 3

STEP 4

Use the quadratic regression feature to find the best fitting quadratic model for the data. Check how well the model fits the data by graphing the quadratic model for the data. model and the data in the same viewing window.



ANSWER

The best-fitting quadratic model is

$y =$

Pumpkin Tossing

At what angle does the pumpkin travel the farthest?

Explain how you found your answer.

ANSWER

Section 4.7 Completing the Square

Solve the equation by finding square roots.

1. $x^2 + 2x + 1 = 9$

2. $x^2 + 6x + 9 = 1$

3. $x^2 - 4x + 4 = 16$

4. $x^2 - 10x + 25 = 4$

5. $x^2 - 14x + 49 = 7$

6. $x^2 + 20x + 100 = 12$

Find the value of c that makes the expression a perfect square trinomial.
Then write the expression as a square of a binomial.

10. $x^2 + 4x + c$

11. $x^2 - 2x + c$

12. $x^2 + 18x + c$

13. $x^2 + 24x + c$

14. $x^2 - 14x + c$

Solve the equation by completing the square.

18. $x^2 - 2x - 2 = 0$

19. $x^2 + 6x + 3 = 0$

20. $x^2 + 8x - 2 = 0$

Write the quadratic function in vertex form. Then identify the vertex.

26. $y = x^2 + 8x + 5$

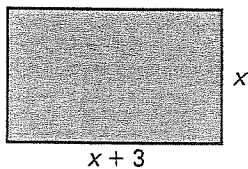
27. $y = x^2 - 12x + 1$

28. $y = x^2 + 4x + 12$

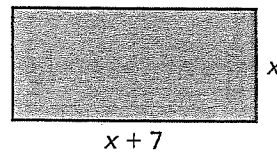
29. $y = x^2 - 10x + 3$

Find the value of x .

30. Area of rectangle = 40



31. Area of rectangle = 78



4.8, Use the Quadratic Formula and the Discriminant

GOAL Solve quadratic equations using the quadratic formula.

Vocabulary

The **quadratic formula**: Let a , b , and c be real numbers where $a \neq 0$. The solutions of the quadratic equation $ax^2 + bx + c = 0$ are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

In the quadratic formula, the expression $b^2 - 4ac$ is called the **discriminant** of the associated equation $ax^2 + bx + c = 0$.

Example 1 Solve an equation with two real solutions

Solve $x^2 + 7x = 6$.

$$x^2 + 7x = 6$$

Original equation

$$x^2 + 7x \text{ _____} = 0$$

Standard form

$$x = \frac{\text{_____} \pm \sqrt{\text{_____} - 4 \text{_____}}}{2 \text{_____}}$$

Quadratic formula

$$x = \frac{\text{_____} \pm \sqrt{\text{_____} - 4 \text{_____}}}{2 \text{_____}}$$

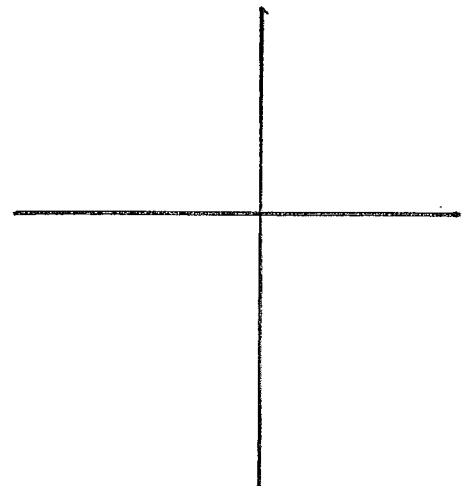
$a = \text{_____}$, $b = \text{_____}$,
 $c = \text{_____}$

$$x = \text{_____}$$

Simplify.

The solutions are $x = \text{_____} \approx \text{_____}$ and

$$x = \text{_____} \approx \text{_____}$$



Example 2 Solve an equation with one real solution

Solve $2x^2 - 8x + 8 = 0$.

Solution

$2x^2 - 8x + 8 = 0$

Original equation

$x = \frac{\pm \sqrt{\quad - 4 \quad}}{2 \quad}$

$a = \quad, b = \quad,$
 $c = \quad$

$x = \quad$

Simplify.

$x = \quad$

Simplify.

The solution is \quad .

Example 3 Solve an equation with imaginary solutions

Solve $-x^2 + 2x = 5$.

Solution

$-x^2 + 2x = 5$

Original equation

$-x^2 + 2x \quad = 0$

Standard form

$x = \frac{\pm \sqrt{\quad - 4 \quad}}{2 \quad}$

$a = \quad, b = \quad,$
 $c = \quad$

$x = \quad$

Simplify.

$x = \quad$

Rewrite using the
imaginary unit i .

$x = \quad$

Simplify.

The solutions are \quad and \quad .

Use the quadratic formula to solve the equation.

1. $x^2 + 4x = 2$

2. $2x^2 - 8x = 1$

3. $4x^2 + 2x = -2x - 1$

4. $16x^2 - 20x = 4x - 9$

5. $x^2 - 4x + 5 = 0$

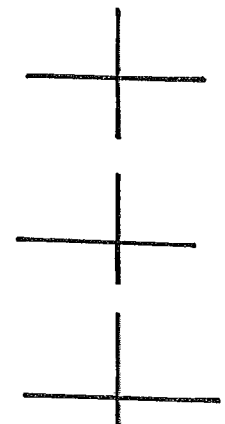
6. $x^2 - x = -7$

USING THE DISCRIMINANT OF $ax^2 + bx + c = 0$

When $b^2 - 4ac > 0$, the equation has _____
_____. The graph has _____ x-intercepts.

When $b^2 - 4ac = 0$, the equation has _____
_____. The graph has _____ x-intercept.

When $b^2 - 4ac < 0$, the equation has _____
_____. The graph has _____ x-intercepts.



Find the discriminant of the quadratic equation and give the number and type of solutions of the equation.

7. $x^2 - 2x - 1 = 0$

8. $x^2 - 12x + 36 = 0$

9. $x^2 + 7x + 14 = 0$

Algebra Two Criteria for the Chapter 4 Test

Students should be able to.....

- Give the vertex and axis of symmetry for quadratic equations in standard form, vertex form, and intercept form (4.1/4.2)
- Put an equation in standard form when the given equation is in vertex or intercept form (4.2)
- Factor (4.3/4.4)
- Simplify radicals (remember no decimals and no radicals in the denominator) (4.5)
- Multiply and divide radicals (4.5)
- Use the conjugate to simplify a radical (4.5)
- Write the expression of a complex number in standard form ($a + bi$) (4.6)
- Solve a quadratic equation by factoring and using the zero product property (4.3/4.4)
- Solve quadratic equations by square roots (4.5)
- Solve quadratic equations by completing the square (4.7)
- Solve quadratic equations by using the quadratic formula (4.8)
- Find the discriminant and give the number and type of solutions (4.8)
- Write the quadratic equation in vertex form by completing the square and then identify the vertex (4.7)
- Find the time it takes for an object to reach the ground when dropped off a cliff using the given formula $y = -16t^2 + h_0$ (4.5)

Section 7.8 Geysers Application**Geysers**

EARTH SCIENCE A geyser is a deep, narrow hole filled with water. The water near the bottom of the hole is heated by the hot rocks found in the Earth. Gradually the heat at the bottom reaches far above the boiling point of water, causing steam to form. The steam begins to build up pressure because it cannot escape into the air. More water turns to steam and the pressure continues to grow until the steam pushes up with a tremendous force. This force causes the water near the top of the hole to be thrown into the air.

Geysers are sometimes compared to volcanoes. They are similar in that the inner heat of the Earth causes them to erupt. They are different in that volcanoes erupt with molten lava while geysers shoot forth hot water containing dissolved mineral matter.

In Exercises 1–3, use the following information.

A geyser sends a blast of boiling water high into the air. During the eruption, the height h (in feet) of the water t seconds after being forced out from the ground could be modeled by $h = -16t^2 + 70t$.

1. What is the initial velocity of the boiling water?
2. What is the maximum height of the boiling water?
3. How long is the boiling water in the air?

In Exercises 4–6, use the following information.

Old Faithful in Yellowstone Park is probably the world's most famous geyser. Old Faithful sends a stream of boiling water into the air. During the eruption, the height h (in feet) of the water t seconds after being forced out from the ground could be modeled by $h = -16t^2 + 150t$.

4. What is the initial velocity of the boiling water?
5. What is the maximum height of the boiling water?
6. How long is the boiling water in the air?

Section 4.8 Graphing Calculator Activity**GOAL** To determine the number of real solutions of a quadratic equation

For the quadratic equation $ax^2 + bx + c = 0$, you can predict the equation's number of real solutions by finding the value of $b^2 - 4ac$.

Activity

- 1 For the following equations, calculate $b^2 - 4ac$.
 $2x^2 + x + 1 = 0$
 $2x^2 - 6x + 3 = 0$
 $x^2 + 4x + 4 = 0$
- 2 Use a graphing calculator to graph the three equations in Step 1.
- 3 Each x -intercept on the graph is a real solution of the related equation. Using your results from Step 1, make a conjecture as to the number of real solutions of an equation $ax^2 + bx + c = 0$ if
 - a. $b^2 - 4ac$ is zero.
 - b. $b^2 - 4ac$ is positive.
 - c. $b^2 - 4ac$ is negative.

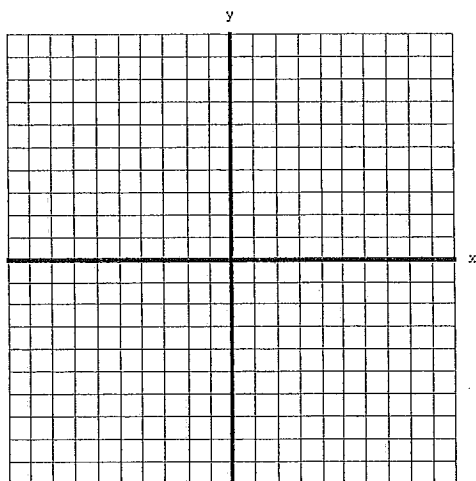
Exercises

1. By calculating $b^2 - 4ac$, predict the number of real solutions of the following equations.
 - a. $x^2 - 5x - 14 = 0$
 - b. $x^2 - 2x - 4 = 0$
 - c. $7x^2 + 2x + 9 = 0$
 - d. $3x^2 + 6x + 3 = 0$
2. Check your answer to Exercise 1 with a graphing calculator.
3. By calculating $b^2 - 4ac$, predict the number of real solutions of the following equations.
 - a. $x^2 - 3x = 15$
 - b. $x^2 = 6x + 9$
 - c. $5x^2 = -2x - 7$
 - d. $6x^2 = -5x - 1$
4. Check your answer to Exercise 3 with a graphing calculator.

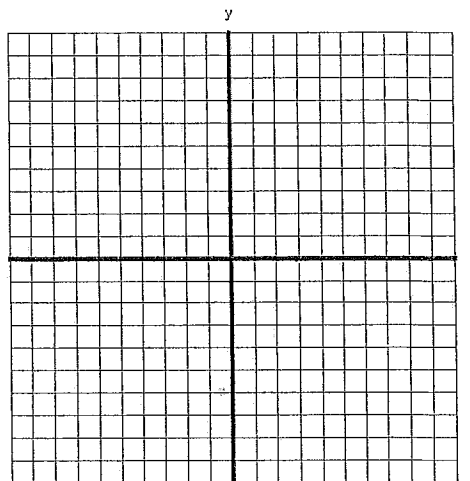
Chapter 4 Review

Graph the function. Label the vertex and the axis of symmetry.

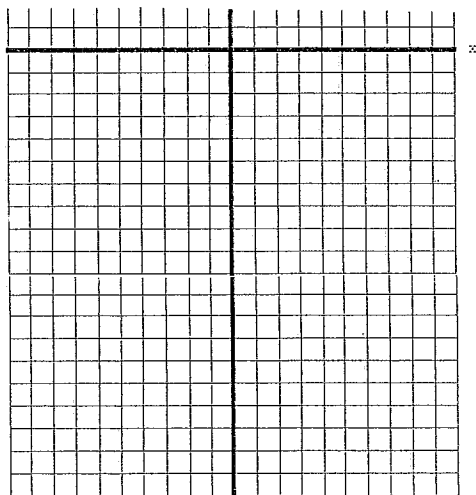
1. $y = 2x^2 + 6x - 1$



2. $y = -3(x - 3)^2 + 5$



3. $y = (x - 3)(x + 6)$



Write the quadratic function in standard form.

4. $y = (x - 2)(x + 6)$

5. $y = 2(x + 4)^2 - 3$

Factor the expression completely.

6. $x^2 - 3x - 10$

7. $-2x^2 + 6x + 56$

8. $9x^2 - 13x + 4$

9. $2x^2 + 9x - 18$

Simplify the expression completely.

10. $\sqrt{20}$

11. $\sqrt{5}\sqrt{10}$

12. $\frac{2 - \sqrt{3}}{3 + \sqrt{5}}$

13. $\sqrt{\frac{7}{11}}$

14. $\sqrt{\frac{35}{36}}$

15. $\sqrt{3}\sqrt{27}$

Write the expression as a complex number in standard form. $a + bi$

16. $(4 - i) + (6 - 3i) - (2 - 4i)$

17. $7 - (2 + 5i) + (5 - 4i)$

Solve the equation by factoring.

18. $x^2 - 3x - 40 = 0$

19. $10x^2 + 62x = -12$

Solve the equation by square roots.

20. $5x^2 = 80$

21. $-5(x-3)^2 = 10$

Solve the equation by completing the square.

22. $x^2 - 12x + 4 = 0$

23. $2x^2 + 8x + 14 = 0$

Solve the equation using the quadratic formula.

24. $x^2 + 4x - 3 = 0$

25. $6x^2 - 8x = -3$

26. Find the discriminant of $9x^2 = -6x - 1$ and give the number and type of solutions.

27 . Write the quadratic in vertex form. Then identify the vertex.

$$y = x^2 + 14x + 45$$

28. An object is dropped off a cliff at height of 50 feet.

Using the free fall model $h = -16t^2 + h_0$ find how long the object is in the air.