Algebra II

Name______________________
Date______________ Hr________

5.1, Use Properties of Exponents

GOAL **Simplify expressions involving powers.**

Vocabulary

A number is expressed in **scientific notation** if it is in the form $c \times 10^n$ where $1 \le c < 10$ and n is an integer.

PROPERTIES OF EXPONENTS

Let a and b be real numbers and let m and n be integers.

Product of Powers Property	$a^m \cdot a^n = a^{\_\_\_\_}$
Power of a Power Property	$(a^m)^n = a^{\_\_\_\_}$
Power of a Product Property	$(ab)^m = a^{\_\_\_}b^{\_\_\_}$
Negative Exponent Property	$a^{-m} = \frac{\quad}{\_\_\_\_}, a \neq 0$
Zero Exponent Property	$a^0 = \_\_\_\_, a \neq 0$
Quotient of Powers Property	$\frac{a^m}{a^n} = a^{\_\_\_\_}, a \neq 0$
Power of a Quotient Property	$\left(\frac{a}{b}\right)^m = \frac{\quad}{\_\_\_\_}, b \neq 0$

Example 1 ***Evaluate numerical expressions***

a. $(6^2)^3 = 6^{\_\_\_\_} = 6^{\_\_\_} = \_\_\_\_\_\_$

b. $\frac{4^5}{4^3} = 4^{\_\_\_\_} = 4^{\_\_\_} = \_\_\_\_$

c. $7^{-4} = \frac{\quad}{\_\_\_\_} = \frac{\quad}{\_\_\_\_\_}$

Example 2 *Use scientific notation in real life*

Iceland Iceland covers about 1.03×10^5 square kilometers and has approximately 2.94×10^5 people. About how many people are there per square kilometer?

Solution

$\frac{\text{Population}}{\text{Land area}} = \frac{\_\_\_\_}{\_\_\_\_}$ **Divide population by land area.**

$= \frac{\_\_\_\_}{\_\_\_\_}$ **Quotient of powers property**

$\approx$ ____ **Use a calculator.**

$=$ ____ **Zero exponent property**

There are about ___ people per square kilometer.

Example 3 *Simplify expressions*

a. $\frac{(x^5y^2)^3}{x^{15}y^8} = \frac{\square}{x^{15}y^8}$ **Power of a product property**

$= \frac{\square}{x^{15}y^8}$ **Power of a power property**

$=$ ________ **Quotient of powers property**

$=$ ____ **Simplify exponents.**

$=$ ____ **Zero exponent property**

$= \frac{\_\_\_\_}{\_\_\_\_}$ **Negative exponent property**

b. $\left(\frac{a^{-4}}{b^2}\right)^2 = \frac{\square}{\square}$ **Power of a quotient property**

$= \frac{\square}{\square}$ **Power of a power property**

$= \frac{\square}{\square}$ **Negative exponent property**

Evaluate the expression.

1. $(2^2 \cdot 5)^3$
2. $7^3 \cdot 7^{-1}$
3. $(8^0 \cdot 6^{-2})^{-1}$
4. $\left(\frac{9^6}{9^4}\right)^3$
5. $\left(\frac{3x}{z^2}\right)^0$
6. $t^7 t^2 t^{-8}$
7. $(k^{-3}m^4)^{-2}$
8. $\left(\frac{f^5}{g^{-2}}\right)^{-3}$

Example 4 ***Compare real-life volumes***

Beach Ball The radius of a beach ball is about 5.6 times greater than the radius of a baseball. How many times as great as the baseball's volume is the beach ball's volume?

Solution

Let r represent the radius of the baseball.

$$\frac{\text{Beach ball's volume}}{\text{Baseball's volume}} = \frac{\frac{4}{3}\pi(\boxed{})^3}{\frac{4}{3}\pi r^3}$$ **The volume of a sphere is $\frac{4}{3}\pi r^3$.**

$$= \frac{\frac{4}{3}\pi\boxed{}}{\frac{4}{3}\pi r^3}$$ **Power of a product property**

$= \_\_\_\_\_\_$ **Quotient of powers**

$= \_\_\_\_\_\_$ **Zero exponent property**

$\approx \_\_\_\_\_\_$ **Approximate power.**

The beach ball's volume is about ______ times as great as the baseball's volume.

9. Rework Example 4 where the radius of a volleyball is about 3 times the radius of a baseball.

Assignment 5-1 Pages 333-334, #4-32 even, 49, 50

5-1 Advanced Algebra

Evaluate the expression.

21) $\left(\frac{3}{7}\right)^3$

23) $11^{-2} \cdot 11^0$

25) $\left(\frac{1}{8}\right)^{-4}$

27) $\frac{2^2}{2^{-9}}$

29) $6^0 \cdot 6^3 \cdot 6^{-4}$

31) $\left(\left(\frac{2}{5}\right)^{-3}\right)^2$

Simplify the expression.

33) $(2^3x^2)^5$

37) $(x^4y^7)^{-3}$

41) $\frac{x^{-1}y}{xy^{-2}}$

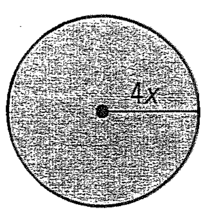
43) $\frac{2x^2y}{6xy^{-1}}$

45) $\frac{xy^9}{3y^{-2}} \cdot \frac{-7y}{21x^5}$

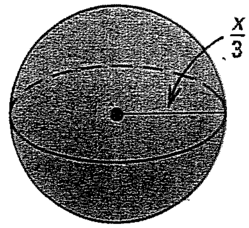
47) $\frac{12xy}{7x^4} \cdot \frac{7x^5y^2}{4y}$

Write an expression for the area or volume of the figure in terms of x.

49) $A = \pi r^2$



51) $V = \frac{4}{3}\pi r^3$



Write the expression as a complex number in standard form.

79) $(-5 + 3i) - (-2 - i)$

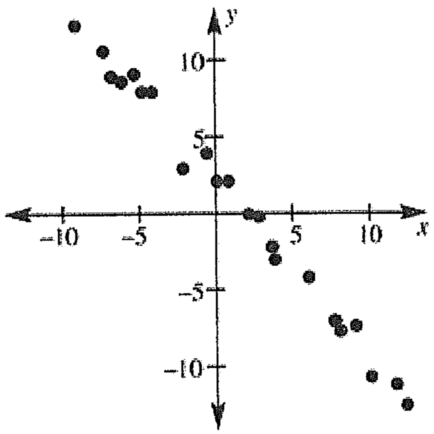
81) $-i(7 + 2i)$

83) $(3 + i)(9 + i)$

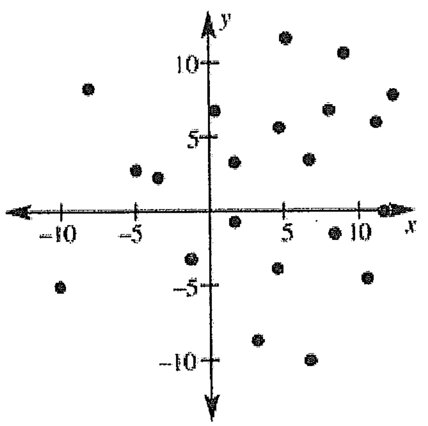
Name: ______________________ Class: __________________ Date: _________ ID: A

Algebra II **2-6**

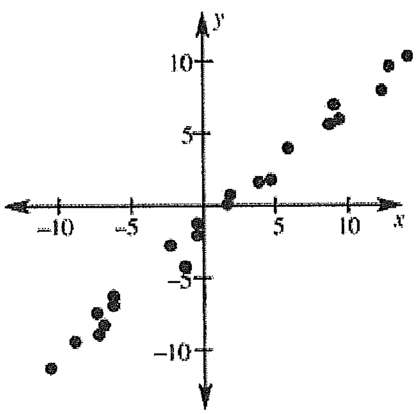
1. For the scatter plot shown, state whether x and y have a *positive correlation*, a *negative correlation*, or *no correlation*.



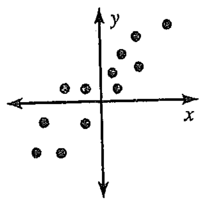
2. For the scatter plot shown, state whether x and y have a *positive correlation*, a *negative correlation*, or *no correlation*.



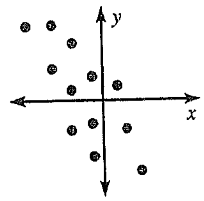
3. For the scatter plot shown, state whether x and y have a *positive correlation*, a *negative correlation*, or *no correlation*.



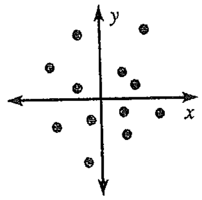
A **scatter plot** is a graph of a set of data pairs (x, y). If y tends to increase as x increases, then the data have a **positive correlation.** If y tends to decrease as x increases, then the data have a **negative correlation.** If the points show no obvious pattern, then the data have *approximately no correlation.*



Positive correlation



Negative correlation



Approximately no correlation

CORRELATION COEFFICIENTS A **correlation coefficient**, denoted by r, is a number from -1 to 1 that measures how well a line fits a set of data pairs (x, y). If r is near 1, the points lie close to a line with positive slope. If r is near -1, the points lie close to a line with negative slope. If r is near 0, the points do not lie close to any line.

$r = -1$

Points lie near line with a negative slope.

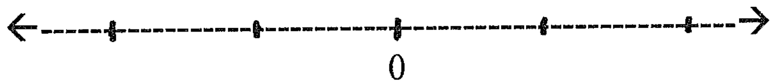
$r = 0$

Points do not lie near any line.

$r = 1$

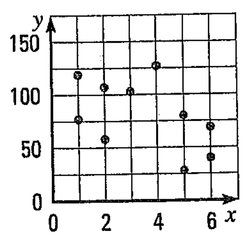
Points lie near line with positive slope.

Plot the Pearson r on the number line and identify strong and weak areas.

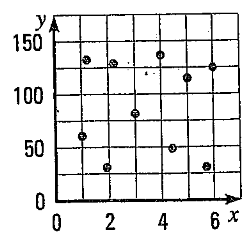


Tell whether the correlation coefficient for the data is closest to -1, -0.5, 0, 0.5, or 1.

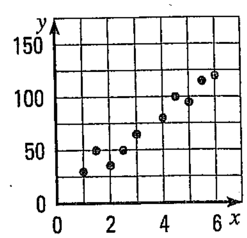
a.



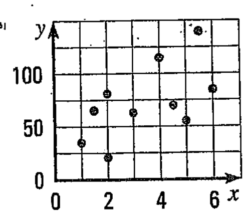
b.



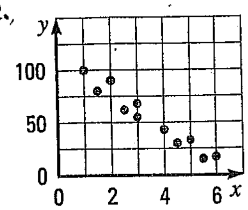
c.



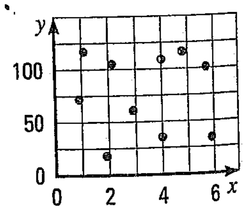
d.



e.



f.



Algebra II

Name______________________
Date______________ Hr________

5.2 Evaluate and Graph Polynomial Functions

GOAL **Evaluate and graph other polynomial functions.**

Vocabulary

A **polynomial** is monomial or a sum of monomials.

A **polynomial function** is a function of the form $f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$ where $a^n \neq 0$, the exponents are all whole numbers, and the coefficients are all real numbers.

Synthetic substitution is another way to evaluate a polynomial function, involving fewer operations than direct substitution.

The **end behavior** of a polynomial function's graph is the behavior of the graph as x approaches positive infinity or negative infinity.

Example 1 *Identify polynomial functions*

Decide whether the function is a polynomial function. If so, write it in standard form and state its degree, type, and leading coefficient.

a. $f(x) = 3x^3 + 4x^{2.5} - 6x^2$ **b.** $f(x) = x^2 + 3.7x + 9x^4$

Solution

a. The function ________ a polynomial function because the term _______ has an exponent that is ________ ____________________.

b. The function ____ a polynomial function written as ________________________ in its standard form. It has degree ___ (__________) and a leading coefficient of ___.

linear:

quadratic:

cubic:

quartic:

If the function is a polynomial function, write it in standard form and state its degree, type, and leading coefficient.

1. $g(x) = \pi x + 7$ **2.** $s(x) = 2x^2 + x^{-1}$ **3.** $d(x) = 3\pi x^2$

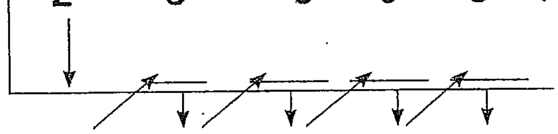
degree:
type:
leading coefficient:

Example 2 ***Evaluate by synthetic substitution***

Use synthetic substitution to evaluate $f(x) = 2x^4 + 3x^3 - 6x^2 + 3$ when $x = 2$.

Write the coefficients of $f(x)$ in order of ____________ exponents. Write the value of x to the left. Bring down the leading coefficient. Multiply the leading coefficient by ___ and write the product under the second coefficient. ______. Multiply the previous sum by ___ and write the product under the second coefficient. Add. Repeat for all of the remaining coefficients.

___ | 2   3   −6   0   3 ← coefficients



___   ___   ___   ___   ___

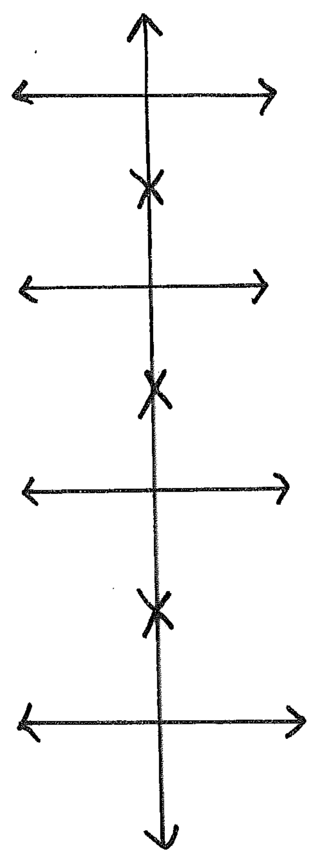
$f(2) =$ ____

4. Evaluate $g(x) = -4x^2 + 6$ when $x = 3$ using direct substitution. Check with synthetic substitution.

END BEHAVIOR OF POLYNOMIAL FUNCTIONS

For the graph of
$f(x) = a_n x^n + a_{n-1}x^{n-1} + \dots + a_1 x + a_0$:

- If $a_n > 0$ and n is odd, then $f(x) \to$ _____ as $x \to -\infty$ and $f(x) \to$ _____ as $x \to +\infty$.
- If $a_n < 0$ and n is odd, then $f(x) \to$ _____ as $x \to -\infty$ and $f(x) \to$ _____ as $x \to +\infty$.
- If $a_n > 0$ and n is even, then $f(x) \to$ _____ as $x \to -\infty$ and $f(x) \to$ _____ as $x \to +\infty$.
- If $a_n < 0$ and n is even, then $f(x) \to$ _____ as $x \to -\infty$ and $f(x) \to$ _____ as $x \to +\infty$.



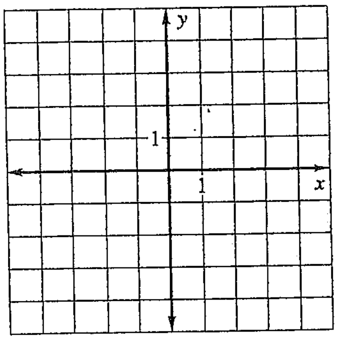
Example 3 ***Graph polynomial functions***

Graph $f(x) = -x^3 + 2x^2 + 2x - 1$.

Solution

Make a table of values and plot the corresponding points. Connect the points with a smooth curve and check the end behavior.

x	-3	-2	-1	0	1	2	3
$f(x)$	___	___	___	___	___	___	___

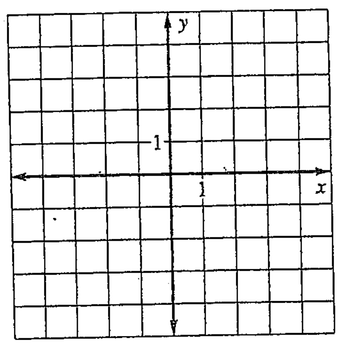


The degree is _____ and the leading coefficient is __________, so $f(x) \to$ _____ as $x \to -\infty$ and $f(x) \to$ _____ as $x \to +\infty$.

Ex. 4. $f(x) = -x^4 + 3x^3 + x^2 - 4x - 1$.

x	y
-3	
-2	
-1	
0	
1	
2	
3	

Graph $f(x)$.



Evaluate $f(x)$ for $x = -2$ using synthetic substitution.

Describe the end behavior of the graph $f(x) \to$ as $x \to -\infty$

$f(x) \to$ as $x \to +\infty$

Assignment 5-2 Pages 341-343, #3-8 all, 10-14 even, 15-20 all, 54

5-2 Use synthetic substitution to evaluate the polynomial function for the given value of x.

1). $f(x) = x^3 + 5x^2 + 4x + 6, x = 2$

2) $f(x) = 2x^3 + x^4 + 5x^2 - x, x = -3$

3) $f(x) = x^3 - x^5 + 3, x = -1$

4) $f(x) = 5x^3 - 4x^2 - 2, x = 0$

Polynomial Functions

Degree	Type	Standard form
0	Constant	$f(x) = a_0$
1	Linear	$f(x) = a_1x + a_0$
2	Quadratic	$f(x) = a_2x^2 + a_1x + a_0$
3	Cubic	$f(x) = a_3x^3 + a_2x^2 + a_1x + a_0$
4	Quartic	$f(x) = a_4x^4 + a_3x^3 + a_2x^2 + a_1x + a_0$

5-1 Notes

Practice B

For use with pages 323–328

Use the properties of exponents to evaluate the expression.

1. $(3^4)(3^{-2})$

2. $(5^2)^3$

3. $\left(\frac{2}{3}\right)^3$

4. $\frac{8^4}{8^6}$

5. $(7^6)(7^{-6})$

6. $\frac{4 \cdot 4^3}{4^6}$

7. $\frac{(3^2)^5}{3^8}$

8. $\left(\frac{1}{2}\right)^{-4}$

9. $\frac{5^6}{(5^3)^2}$

Simplify the expression.

10. $x^3 \cdot x^2$

11. $\frac{2y^3}{y^5}$

12. $(3x)^2$

13. $\left(\frac{y}{2}\right)^3$

14. $(4x^3)^4$

15. x^0y^{-2}

16. $\frac{5x^2y}{2x^{-1}y^3}$

17. $\frac{-3xy}{9x^3y^{-4}}$

18. $\frac{(3x)^2}{6x^5}$

Algebra II Practice A 5-1 Name ____________________

Use the properties of exponents to evaluate the expression.

1. $3^4 \cdot 3^5$
2. $2^6 \cdot 2^2$
3. $4^3 \cdot 4^2$
4. $4^{-1} \cdot 4^4$
5. $2^{-5} \cdot 2^3$
6. $5^{-7} \cdot 5^8$
7. $6^{-2} \cdot 6^{-1}$
8. $3^{-2} \cdot 3^{-3}$
9. $2^{-4} \cdot 2^{-3}$
10. $(-3)^2(-3)^3$
11. $(-5)^{-6}(-5)^8$
12. $(-2)^{-3}(-2)^{-2}$
13. $\frac{5^4}{5^2}$
14. $\frac{7^6}{7^9}$
15. $\frac{3^5}{3^5}$
16. $\frac{(-2)^8}{(-2)^3}$
17. $\frac{(-3)^3}{(-3)^4}$
18. $(5^2)^3$
19. $(2^4)^2$
20. $(3^2)^3$
21. $\left(\frac{1}{3}\right)^4$
22. $\left(\frac{3}{2}\right)^3$
23. $\left(-\frac{2}{5}\right)^2$
24. $\left(-\frac{1}{4}\right)^3$
25. 13^0
26. $\left(\frac{4}{5}\right)^0$
27. 3^{-2}
28. 4^{-3}
29. $\left(\frac{3}{4}\right)^{-2}$
30. $\left(\frac{2}{3}\right)^{-4}$

Simplify the expression.

31. $x^3 \cdot x^5$
32. $x^4 \cdot x^8$
33. $(x^4)^6$
34. $(3x)^3$
35. $\left(\frac{x}{2}\right)^4$
36. $\frac{x^7}{x^2}$
37. $\frac{x^3}{x^9}$
38. $\left(\frac{3}{x}\right)^2$
39. $\left(\frac{x}{4}\right)^{-2}$

Algebra 2

Name ______________________ Date ______________

LESSON 5.2

Practice B

For use with pages 336–345

Decide whether the function is a polynomial function. If it is, write the function in standard form and state the degree, type, and leading coefficient.

1. $f(x) = 7 - 2x$
2. $g(x) = 2x - x^3 + 8$
3. $h(x) = x^4 - x^{-3}$

Use direct substitution to evaluate the polynomial function for the given value of *x*.

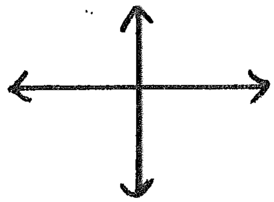
4. $f(x) = 6x^4 - x^3 + 3x^2 - 5x + 9;\ x = -1$
5. $g(x) = 7x - x^4 + 1;\ x = -4$

Use synthetic substitution to evaluate the polynomial function for the given value of *x*.

6. $f(x) = 7x^4 - 3x^3 + x^2 + 5x - 9;\ x = 2$
7. $g(x) = x^3 - 8x + 6;\ x = -3$

Describe the end behavior of the graph of the polynomial function by completing these statements: $f(x) \to$ __?__ as $x \to -\infty$ and $f(x) \to$ __?__ as $x \to +\infty$.

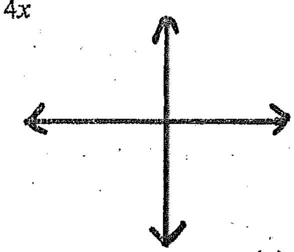
8. $f(x) = -5x^3$



f(x)→ as x→-∞

f(x)→ as x→+∞

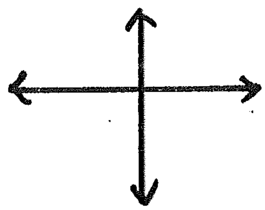
9. $f(x) = 2x^5 - 7x^2 - 4x$



f(x)→ as x→-∞

f(x)→ as x→+∞

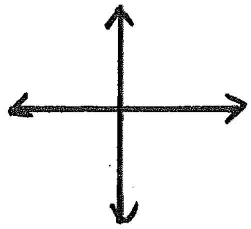
10. $f(x) = 2x^8 + 9x^7 + 10$



f(x)→ as x→-∞

f(x)→ as x→+∞

11. $f(x) = -12x^6 - 2x + 5$

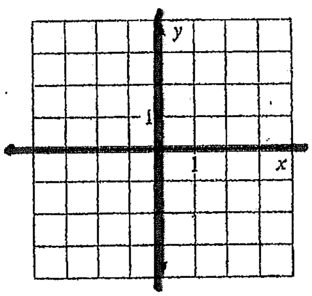


f(x)→ as x→-∞

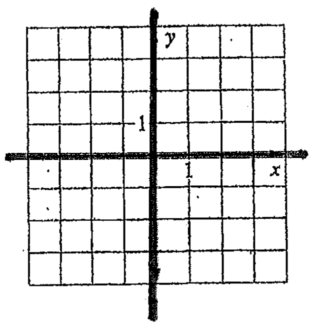
f(x)→ as x→+∞

Graph the polynomial function.

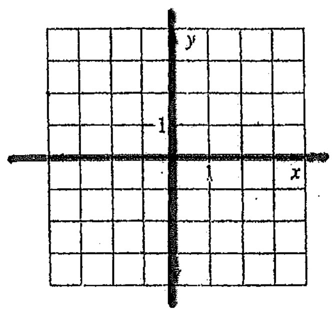
12. $f(x) = -x^3 - 2$



13. $g(x) = x^4 + 2x$



14. $h(x) = -x^4 + 2x^3 - 5x + 1$



Evaluate the power.   Notes

1) 3^2 2) $(-3)^2$ 3) -3^2 4) $3^{-2} \cdot 3^5$

LESSON 5.1 **Practice B**
For use with pages 330–335

Evaluate the expression.

1. $2^5 \cdot 2^3$
2. $(-7)^2(-7)$
3. $4^{-6} \cdot 4^{-1}$
4. $(5^{-2})^2$
5. $\frac{4^{-7}}{4^{-3}}$
6. $\frac{8^{-4}}{8^2}$
7. $\left(\frac{2}{3}\right)^3$
8. $\left(\frac{4}{5}\right)^{-3}$

Write the answer in scientific notation.

9. $(6.1 \times 10^5)(2.2 \times 10^6)$
10. $(2.6 \times 10^{-7})(1.3 \times 10^2)$
11. $(3.4 \times 10^{-1})(3.1 \times 10^{-2})$
12. $(5.8 \times 10^{-7})(8.1 \times 10^{12})$
13. $(4.5 \times 10^4)^2$
14. $(3.7 \times 10^{-5})^2$
15. $(7.2 \times 10^{-3})^3$
16. $\frac{9.9 \times 10^9}{1.5 \times 10^8}$
17. $\frac{8.4 \times 10^{-6}}{2.4 \times 10^9}$

Simplify the expression.

18. $\frac{x^8}{x^4}$
19. $\frac{y^4}{y^{-7}}$
20. $(3^2s^3)^6$
21. $(4^0w^2)^{-5}$
22. $(y^4z^2)(y^{-3}z^{-5})$
23. $(2m^3n^{-1})(8m^4n^{-2})$
24. $(7c^7d^2)^{-2}$
25. $(5g^4h^{-3})^{-3}$
26. $\frac{x^5y^{-8}}{x^5y^{-6}}$
27. $\frac{16q^0r^{-6}}{4q^{-3}r^{-7}}$
28. $\frac{12a^{-3}b^9}{21a^2b^{-5}}$
29. $\frac{8e^{-4}f^{-2}}{18ef^{-5}}$

Algebra II

Name ______________________

Date ______________ Hour ______

LESSON 5.1

Practice A

For use with pages 330–335

Evaluate the power.

1. 3^2
2. 5^3
3. 2^5
4. 4^4
5. 9^0
6. 2^{-1}
7. 7^{-2}
8. 10^{-6}

Evaluate the expression.

9. $4^2 \cdot 4^3$
10. $(-3)^4(-3)$
11. $(5^2)^3$
12. $(7^0)^5$
13. $2^0 \cdot 2^{-5}$
14. $\frac{3^7}{3^4}$
15. $(10^3)^3$
16. $\left(\frac{5}{6}\right)^2$
17. $\frac{(-5)^6}{-5}$
18. $\frac{8^2}{8^3}$
19. $\frac{9^2}{9^{-2}}$
20. $\left(\frac{1}{2}\right)^{-5}$

Write the number in scientific notation.

21. 527,000
22. 0.0000526
23. 0.0023
24. 5,983,000,000,000
25. 17,600,000,000,000,000
26. 0.0000007

Write the answer in scientific notation.

27. $(3.2 \times 10^4)(1.5 \times 10^5)$
28. $(5.7 \times 10^{-6})(6.2 \times 10^8)$
29. $(2.8 \times 10^3)^2$
30. $(4.3 \times 10^2)^2$
31. $\frac{8.4 \times 10^{10}}{1.4 \times 10^8}$
32. $\frac{3.6 \times 10^{-5}}{4.8 \times 10^{-7}}$

Simplify the expression.

33. $b^4 \cdot b^2$
34. $x^{-3} \cdot x^5$
35. $(s^7)^2$
36. $(5y)^2$
37. $\frac{z^9}{z^5}$
38. $\frac{m^2}{m^6}$
39. $\left(\frac{x}{3}\right)^3$
40. $\left(\frac{n}{4}\right)^{-2}$

Algebra II

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5.3, Add, Subtract, and Multiply Polynomials

Goal • Add, subtract, and multiply polynomials.

Example 1 ***Add polynomials vertically and horizontally***

a. $\begin{array}{r} 3x^3 - 2x^2 + 4x - 6 \\ + \; x^3 - 5x^2 \quad\quad + 3 \\ \hline \end{array}$

____

b. $(2y^3 + 7y^2 - 6y) + (-4y^2 + 3y - 9)$

= ______________________________

= ____________________

Example 2 ***Subtract polynomials vertically and horizontally***

a.

$\begin{array}{r} 7x^3 - 6x^2 - 3x + 7 \\ -\; (6x^3 + 3x^2 - 7x + 5) \\ \hline \end{array}$ → $\begin{array}{r} 7x^3 - 6x^2 - 3x + 7 \\ \text{______________________} \\ \text{____________________} \end{array}$

b. $(8x^2 - 5x + 11) - (12x^2 - 9x - 3)$

$= 8x^2 - 5x + 11$ ____________________

= ____________________

Find the sum or difference.

1. $(4x^3 - 2x^2 + 5) + (-x^3 - x^2 + 4x - 2)$

2. $(9x^2 - 8x + 3) - (2x^2 + x - 4)$

Example 3 *Multiply polynomials vertically and horizontally*

a.

$$\begin{array}{r} 3x^2 - x + 4 \\ \times \qquad x + 2 \\ \hline \end{array}$$

Multiply $3x^2 - x + 4$ by x.

Multiply $3x^2 - x + 4$ by 2.

Combine like terms.

b. $(x - 3)(x^2 + 2x - 5)$

= ____________

Example 4 *Multiply three binomials*

Multiply $(x - 3)(x + 7)(x + 1)$ in a horizontal format.

$(x - 3)(x + 7)(x + 1)$

= (____________)$(x + 1)$

= ____________

Find the product.

3. $(z^2 - 5z + 3)(z - 1)$

4. $(x - 2)(x - 1)(x + 3)$

SPECIAL PRODUCT PATTERNS

	Example
Sum and Difference	**Example**
$(a + b)(b - a) = a^2 - b^2$	$(x + 2)(x - 2) =$ ________
Square of a Binomial	
$(a + b)^2 = a^2 + 2ab + b^2$	$(y + 4)^2$ $=$ ________
$(a - b)^2 = a^2 - 2ab + b^2$	$(3p^2 - 2)^2$ $=$ ________
Cube of a Binomial	
$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$	$(x + 1)^3$ $=$ ________
$(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$	$(r - 3)^3$ $=$ ________

Find the product.

5. $(x + 2)^3$

6. $(7y - 2)^2$

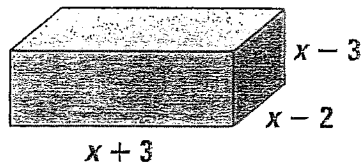
7. $(4d + 3)(4d - 3)$

8. $(2a + 5)^2$

Assignment 5-3 Practice B #1-20all

5-3 Advanced Algebra

12) Write a polynomial model in standard form for the volume of the rectangular prism.



Find the product of the three binomials.

45) $(x + 9)(x - 2)(x - 7)$

51) $(2x + 1)(3x + 1)(x + 4)$

Find the product.

57) $(6 - x^2)^2$

60) $(7y - x)^2$

Solve the equation.

75) $x^2 + 16x + 64 = 0$

77) $2x^2 - 7x - 15 = 0$

Simplify the expression.

83) $x^5 \cdot \frac{1}{x^2}$

85) $-5^{-2}y^0$

87) $\frac{3x^5y^8}{6xy^{-3}}$

Name ______________________________ Date ______________

LESSON 5.3

Practice B

For use with pages 346–352

Find the sum or difference.

1. $(2y^2 - 5y + 1) + (y^2 - y - 4)$

2. $(12x^2 + 8x - 3) - (11x^2 - x + 5)$

3. $(6m^3 - 5) - (m^3 + 4m^2 - 9m - 2)$

4. $(5s^4 - 2s^3 + 9) - (-2s^4 + 8s^2 - s + 2)$

5. $(7q - 3q^3) + (16 - 8q^3 + 5q^2 - q)$

6. $(-4z^4 + 6z - 9) + (11 - z^3 + 3z^2 + z^4)$

7. $(10v^4 - 2v^2 + 6v^3 - 7) - (9 - v + 2v^4)$

8. $(4x^5 + 3x^4 - 5x + 1) - (x^3 + 2x^4 - x^5 + 1)$

Find the product.

9. $2x^3(5x - 1)$

10. $(w - 8)(w - 1)$

11. $(c + 4)(c + 10)$

12. $(g + 9)(g - 2)$

13. $(y - 1)(y^2 + 6y - 2)$

14. $(n + 5)(2n^2 - n - 7)$

15. $(x - 3)^2$

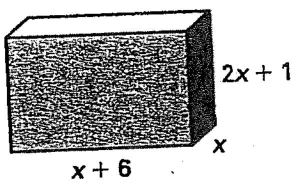
16. $(4t + 1)^2$

17. $(z - 5)^3$

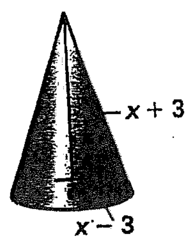
18. $(2f + 1)^3$

Write the volume of the figure as a polynomial in standard form.

19. $V = \ell wh$



20. $V = \frac{1}{3}\pi r^2 h$



Algebra II

Name________________

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5.4, Factor and Solve Polynomial Equations

GOAL **Factor and solve other polynomial equations.**

Vocabulary

A polynomial with two or more terms is a **prime polynomial** if it cannot be written as a product of polynomials of lesser degree using only integer coefficients and constants and if the only common factors of its terms are -1 and 1.

A polynomial is **factored completely** if it is written as a monomial or the product of a monomial and one or more prime polynomials.

For some polynomials you can **factor by grouping** pairs of terms that have a common monomial factor.

An expression of the form $au^2 + bu + c$, where u is any expression in x, is said to be in **quadratic form.**

FACTORING POLYNOMIALS

Definition A polynomial with two or more terms is a prime polynomial if it ________ be written as a product of polynomials of lesser degree using only integer coefficients and constants and if the only common factors of its terms are _____ and ____.

Example $16x^2 - 4x + 8$ ________ a prime polynomial because ___ is a common factor of all its terms.

Definition A polynomial is factored completely if it is written as a monomial or the product of a monomial and one or more ________ polynomials.

Example $(x + 2)(x^2 - 5x + 6)$ is not factored completely because $x^2 - 5x + 6 =$ ______________.

SPECIAL FACTORING PATTERNS

Sum of Two Cubes

$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

Example

$x^3 + 8 = (\qquad)(\underline{\qquad\qquad})$

Difference of Two Cubes

$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

Example

$8x^3 - 1 = (\qquad)(\underline{\qquad\qquad})$

Example 1 *Factor the sum or difference of two cubes*

Factor the polynomial completely.

a. $z^3 - 125 =$ — Difference of two cubes

$= (\underline{\qquad})(\underline{\qquad\qquad})$

b. $81y^4 + 192y = 3y(\underline{\qquad\qquad})$ — Factor common monomial.

Sum of two cubes

$= 3y(\underline{\qquad})(\underline{\qquad\qquad})$

Example 2 *Factor by grouping*

Factor the polynomial $x^3 - 2x^2 - 9x + 18$ completely.

$x^3 - 2x^2 - 9x + 18$

$= \quad(\underline{\qquad}) - \quad(\underline{\qquad})$ — Factor by grouping.

$= \underline{\qquad\qquad}$ — Distributive property

$= \underline{\qquad\qquad\qquad}$ — Difference of two squares

5-4

Example 3 ***Factor polynomials in quadratic form***

Factor completely: (a) $16x^4 - 256$ and (b) $3y^7 - 15y^5 + 18y^3$.

a. $16x^4 - 256 =$ ______________________

$=$ ______________________

$=$ ______________________

b. $3y^7 - 15y^5 + 18y^3 = 3y^3($______________$)$

$=$ ______________________

Factor the polynomial completely.

1. $7x^5 - 56x^2$

2. $128y^6 + 2$

3. $x^3 - 3x^2 - 4x + 12$

4. $y^3 + 7y^2 - 9y - 63$

5. $3b^6 + 6b^4 + 3b^2$

6. $z^8 - 16$

Example 4 ***Solve a polynomial equation***

What are the real-number solutions of the equation $x^4 + 9 = 10x^2$?

$x^4 + 9 = 10x^2$ **Write original equation.**

_______________ $= 0$ **Write in standard form.**

_______________ $= 0$ **Factor trinomial.**

___________________________ $= 0$ **Difference of two squares**

$x =$ _____, $x =$ ____, $x =$ _____, $x =$ ___ **Zero product property**

The solutions are _________________.

Find the real-number solutions of the equation.

7. $x^4 - 3x^2 + 2 = 0$ **8.** $x^5 - 8x^3 = -12x$ **9.** $x^5 - 12x^3 = -27x$

5-4 Factor the Sum or Difference of Cubes

Sum of Two Cubes

$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

Difference of Two Cubes

$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

1) $64x^3 + 1$

2) $54x^3 - 16$

3) $x^3 + 125$

4) $x^3 - 343$

5) $64x^3 - 1$

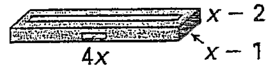
6) $8x^3 + 27$

7) $3x^3 - 24$

8) $1000x^3 - 729$

Solve a polynomial equation

The dimensions (in inches) of a jewelry box are: length $4x$, width $(x - 1)$, and height $(x - 2)$. If the volume of the box is 24 cubic inches, find the dimensions of the box.



Solution

Volume (cubic inches)	=	Length (inches)	•	Width (inches)	•	Height (inches)
↓		↓		↓		↓
24	=	$4x$	•	$(x - 1)$	•	$(x - 2)$

$24 = (4x)(x - 1)(x - 2)$ Write equation.

$0 = 4x^3 - 12x^2 + 8x - 24$ Write in standard form.

$0 = 4x^2(x - 3) + 8(x - 3)$ Factor by grouping.

$0 = (4x^2 + 8)(x - 3)$ Distributive property.

The only real solution is $x = 3$. The jewelry box is 12 inches long, 2 inches wide, and 1 inch high.

10. The dimensions (in inches) of a jewelry box are: length $2x$, width $(x - 1)$, and height $(x - 3)$. If the volume of the box is 24 cubic inches, find the dimensions of the box.

Length

Width

Height

Assignment 5-4 Practice A #1-30 all

Name ______________________ Date ____________

LESSON 5.4

Practice A

For use with pages 353–359

Find the greatest common factor of the terms in the polynomial.

1. $4x^4 + 12x^3$

2. $10y^2 + 4y - 64$

3. $16x^5 - 8x$

4. $32n^5 - 64n^3 + 16n^2$

5. $15p^6 - 5p^4 - 10p^2$

6. $36c^9 + 13$

Match the polynomial with its factorization.

7. $3x^2 + 11x + 6$

8. $x^3 - 4x^2 + 4x - 16$

9. $125x^3 - 216$

10. $2x^7 - 2x^5 - 24x^3$

11. $2x^5 + 4x^4 - 4x^3 - 8x^2$

12. $2x^3 - 32x$

A. $2x^3(x + 2)(x - 2)(x^2 + 3)$

B. $2x(x + 4)(x - 4)$

C. $(3x + 2)(x + 3)$

D. $(x^2 + 4)(x - 4)$

E. $2x^2(x^2 - 2)(x + 2)$

F. $(5x - 6)(25x^2 + 30x + 36)$

Factor the sum or difference of cubes.

13. $s^3 - 1$

14. $q^3 + 1$

15. $x^3 - 27$

16. $a^3 + 125$

17. $h^3 + 64$

18. $8y^3 - 125$

Factor the polynomial by grouping.

19. $x^3 + 2x^2 + 3x + 6$

20. $z^3 - z^2 + 5z - 5$

21. $f^3 + 4f^2 + f + 4$

22. $m^3 - 2m^2 + 4m - 8$

23. $2x^4 - x^3 + 6x - 3$

24. $t^3 - 2t^2 - 9t + 18$

Find the real-number solutions of the equation.

25. $w^2 - 3w = 0$

26. $v^3 + 5v^2 = 0$

27. $x^2 - 5x + 6 = 0$

28. $d^2 - 16 = 0$

29. $10s^3 = 30s^2$

30. $x^3 + x^2 - 9x - 9 = 0$

Algebra II

Name________________________
Date_______________Hr________

Review for Mini Quiz 5.2-5.4

Describe the end behavior of the graph of the polynomial function.

1. $f(x) = -3x^5 - 2x^4 + 3x^3 + 6x$

$f(x) \rightarrow$ __________ $as\ x \rightarrow -\infty$

$f(x) \rightarrow$ __________ $as\ x \rightarrow +\infty$

2. $f(x) = 2x^{20} - 5x^{11} - 4x^9 + 2x^4 + x^2 - 7$

$f(x) \rightarrow$ __________ $as\ x \rightarrow -\infty$

$f(x) \rightarrow$ __________ $as\ x \rightarrow +\infty$

Use synthetic substitution to evaluate the function.

3. $f(x) = 3x^6 - 5x^4 - 3x^3 + x^2 - 4;\ x = 3$

Factor completely.

4. $-5x^4 + 320x$

5. $2x^4 + 10x^3 - 18x^2 - 90x$