

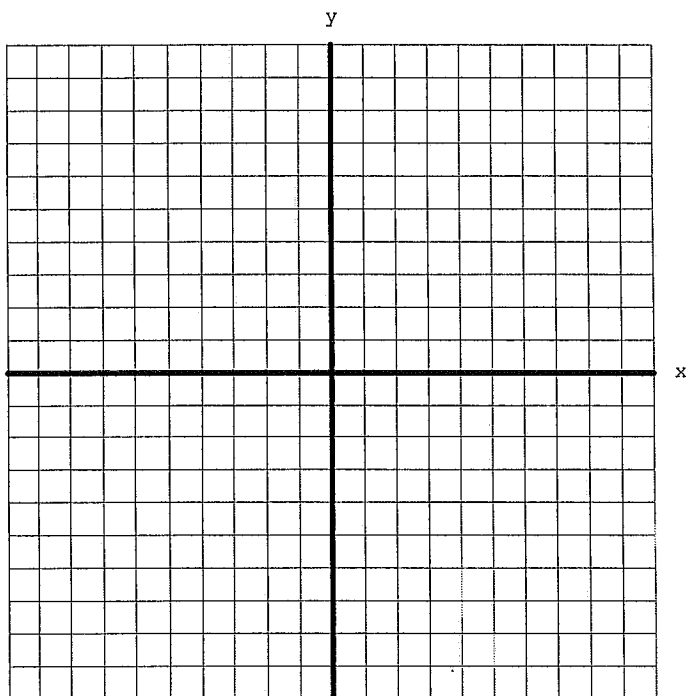
7.1 Graphing Exponential Functions Intro

Basic Function of Exponential Growth and Decay

$$y = a \cdot b^x$$

1. Fill in a table of values. Graph the given function.
2. Describe the graph in terms of its shape (is it continuous?),
 x - and y -values (can x and y be any number?),
 x - y -intercepts,
 increases/decreases?

1. $f(x) = 2^x$



x	y
-2	
-1	
0	
1	
2	
3	

Continuous: yes no

Domain:

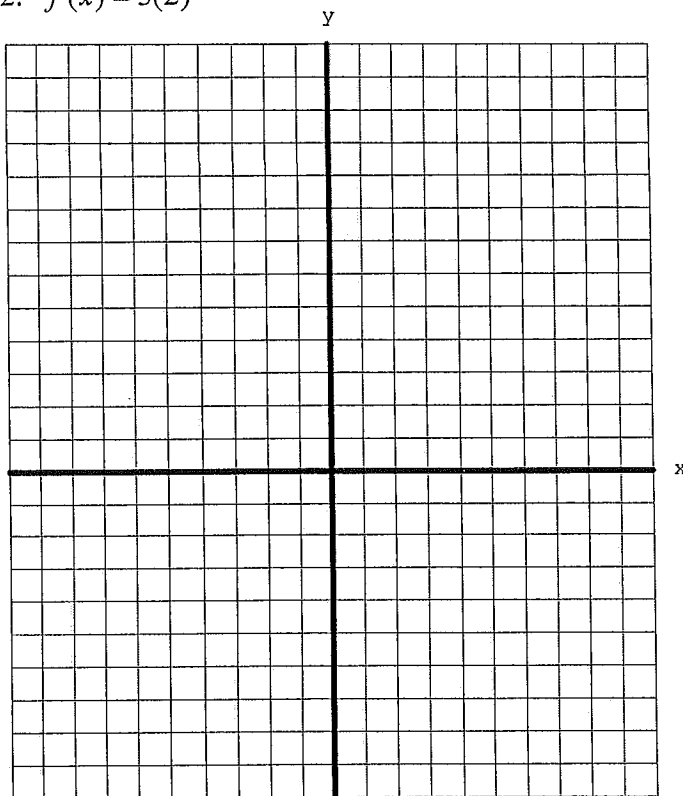
Range:

x -intercept:

y -intercept:

increases decreases

2. $f(x) = 3(2)^x$



x	y
-2	
-1	
0	
1	
2	

Continuous: yes no

Domain:

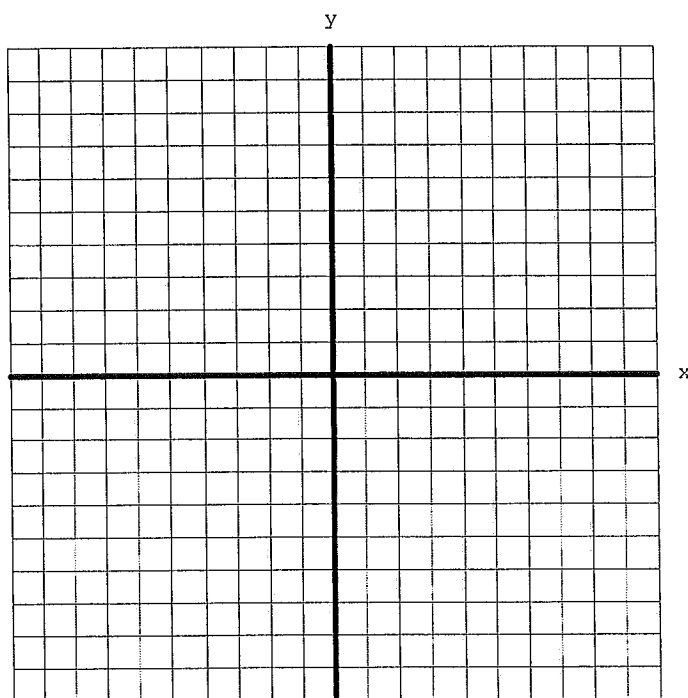
Range:

x-intercept:

y-intercept:

increases decreases

3. $f(x) = \left(\frac{1}{2}\right)^x$



x	y
-3	
-2	
-1	
0	
1	
2	

Continuous: yes no

Domain:

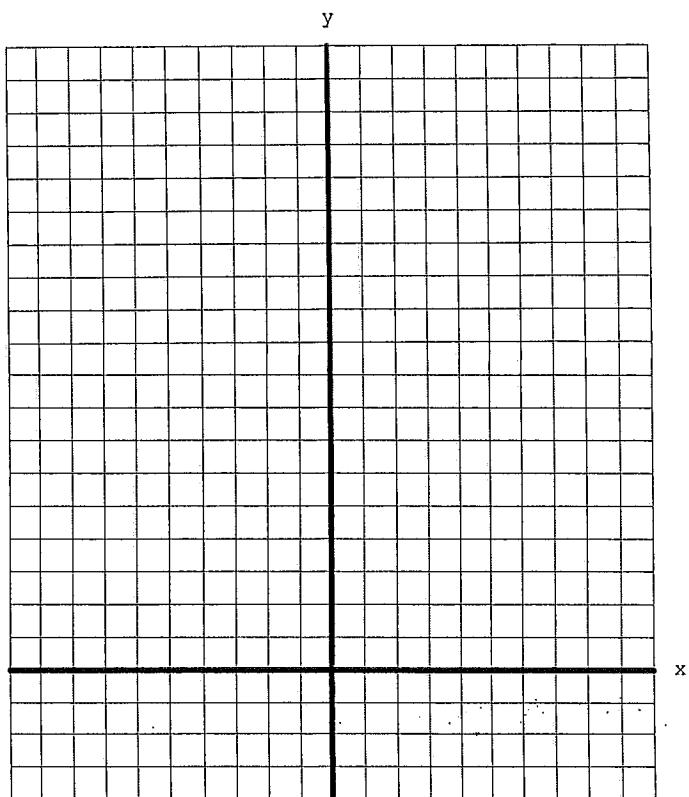
Range:

x-intercept:

y-intercept:

increases decreases

4. $f(x) = 2(3)^x$



x	y
-2	
-1	
0	
1	
2	

Continuous: yes no

Domain:

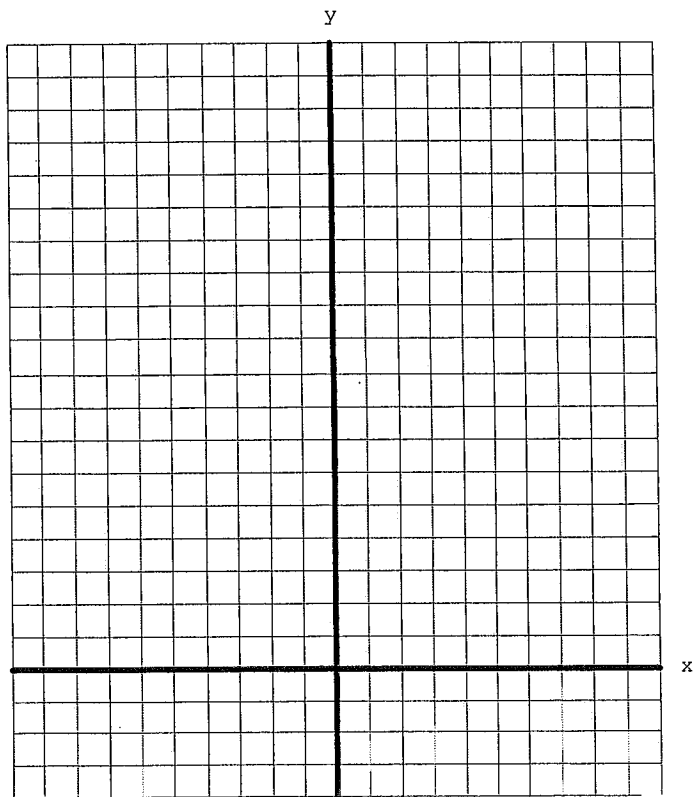
Range:

x-intercept:

y-intercept:

increases decreases

5. $f(x) = 2\left(\frac{1}{3}\right)^x$



x	y
-2	
-1	
0	
1	
2	

Continuous: yes no

Domain:

Range:

x-intercept:

y-intercept:

increases decreases

Section 7.1, Exponential Growth

GOAL

Graph exponential growth functions and use exponential growth functions to model real-life situations

VOCABULARY

An **exponential function** involves the expression b^x where the **base** b is a positive number other than 1. If $a > 0$ and $b > 1$, the function $y = ab^x$ is an exponential growth function.

An **asymptote** is a line that a graph approaches as you move away from the origin. In the exponential growth model $y = a(1 + r)^t$, y is the quantity after t years, a is the initial amount, r is the percent increase expressed as a decimal, and the quantity $1 + r$ is called the **growth factor**.

Compound Interest Consider an initial principal P deposited in an account that pays interest at an annual rate r (expressed as a decimal), compounded n times per year. The amount A in the account after t years can be modeled by this equation: $A = P\left(1 + \frac{r}{n}\right)^{nt}$

EXAMPLE 1

Graphing Exponential Functions

Graph the function (a) $y = -2 \cdot 3^x$ and (b) $y = 2 \cdot 3^x$.

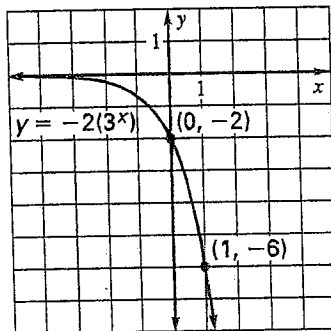
SOLUTION

Begin by plotting two points on the graph. To find these two points, evaluate the function when $x = 0$ and $x = 1$.

a. $y = -2 \cdot 3^0 = -2 \cdot 1 = -2$

$y = -2 \cdot 3^1 = -2 \cdot 3 = -6$

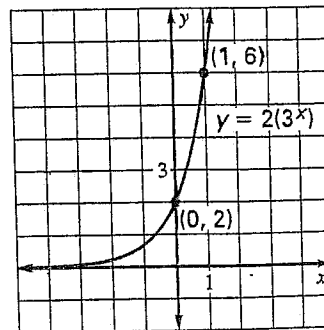
Plot $(0, -2)$ and $(1, -6)$. Then, from left to right, draw a curve that begins just below the x -axis, passes through the two points, and moves down to the right.



b. $y = 2 \cdot 3^0 = 2 \cdot 1 = 2$

$y = 2 \cdot 3^1 = 2 \cdot 3 = 6$

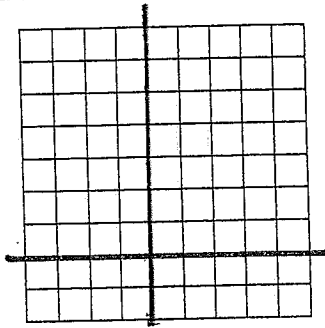
Plot $(0, 2)$ and $(1, 6)$. Then, from left to right, draw a curve that begins just above the x -axis, passes through the two points, and moves up to the right.



Exercises for Example 1

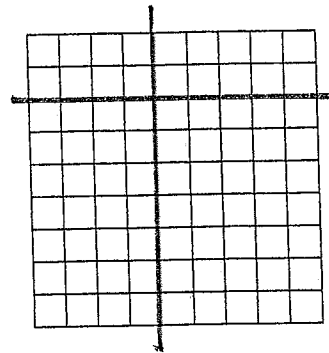
Graph the function.

1. $y = 2^x$



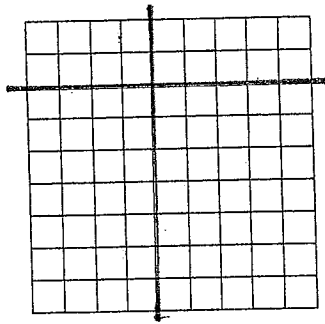
x	y
0	
1	

2. $y = -4^x$



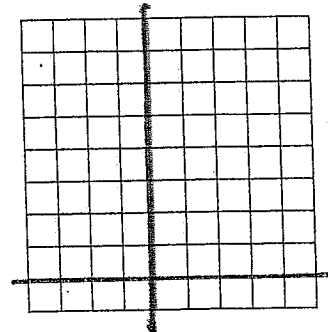
x	y
0	
1	

3. $y = -3 \cdot 2^x$



x	y
0	
1	

4. $y = 4 \cdot 2^x$



x	y
0	
1	

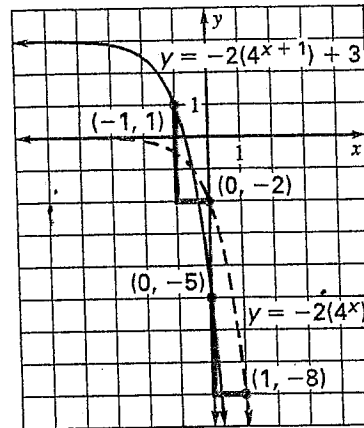
EXAMPLE 2

Graphing a General Exponential Function

Graph $y = -2 \cdot 4^{x+1} + 3$. State the domain and range.

SOLUTION

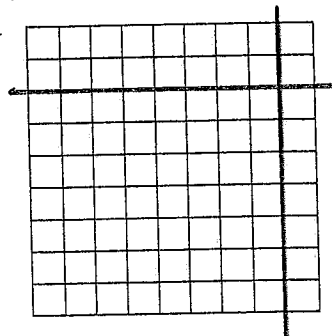
Begin by lightly sketching the graph of $y = -2 \cdot 4^x$, which passes through $(0, -2)$ and $(1, -8)$. Then because $h = -1$ and $k = 3$, translate the graph 1 unit to the left and 3 units up. Notice that the graph passes through $(-1, 1)$ and $(0, -5)$. The graph's asymptote is $y = 3$. The domain is all real numbers and the range is $y < 3$.



Exercises for Example 2

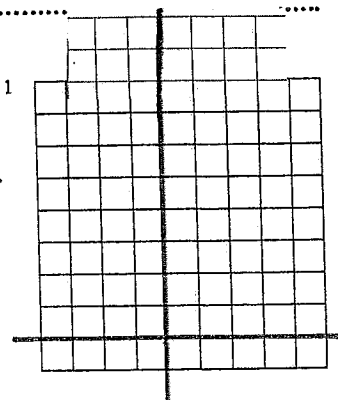
Graph the function. State the domain and range.

5. $y = -3 \cdot 2^{x+4}$



x	y
0	
1	

6. $y = 5 \cdot 2^{x-1}$



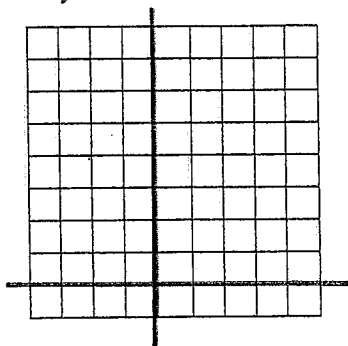
x	y
0	
1	

D:
R:

D:
R:

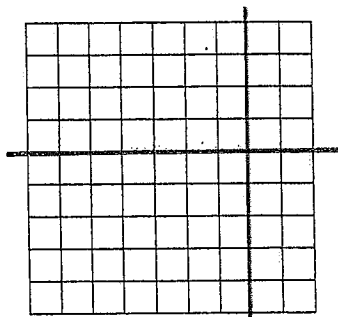
7. $y = 3^{x-2} + 4$

X	Y
0	
1	



D:
R:

8. $y = 4^{x+2} - 3$



D:
R:

X	Y
0	
1	

EXAMPLE 3

Modeling Exponential Growth

A diamond ring was purchased twenty years ago for \$500. The value of the ring increased by 8% each year. What is the value of the ring today?

SOLUTION

The initial amount is $a = 500$, the percent increase expressed in decimal form is $r = 0.08$, and the time in years is $t = 20$.

$y = a(1 + r)^t$	Write exponential growth model.
$= 500(1 + 0.08)^{20}$	Substitute $a = 500$, $r = 0.08$, and $t = 20$.
$= 500 \cdot 1.08^{20}$	Simplify.
≈ 2330.48	Use a calculator.

Compound Interest
 $A = P \left(1 + \frac{r}{n}\right)^{nt}$

The value of the ring today is about \$2330.48.

$n = \# \text{ times per year}$

Exercises for Example 3

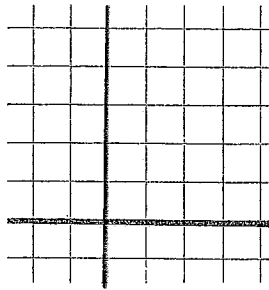
9. A customer purchases a television set for \$800 using a credit card. The interest is charged on any unpaid balance at the rate of 18% per year compounded monthly. If the customer makes no payment for one year, how much is owed at the end of the year?

10. A house was purchased for \$90,000 in 1995. If the value of the home increases by 5% per year, what is it worth in the year 2020?

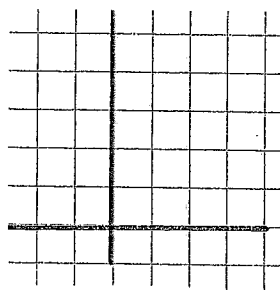
7-1 Advanced Algebra

Identify the y -intercept and the asymptote of the graph of the function.

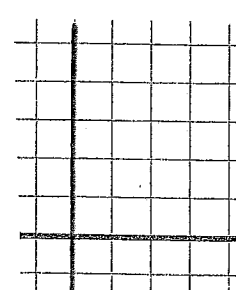
13) $y = 5^x$



15) $y = 4 \cdot 2^x$



17) $y = 3 \cdot 2^{x-1}$



- 43) The amount g (in trillions of cubic feet) of natural gas consumed in the United States from 1940 to 1970 can be modeled by $g = 2.91(1.07)^t$

- a) Initial amount
- b) Growth factor
- c) Annual percent increase

- 46) From 1971 to 1995, the average number n of transistors on a computer chip can be modeled by $n = 2300(1.59)^t$

- a) Initial amount
- b) Growth factor
- c) Annual percent increase

Evaluate the expression.

71) $\left(\frac{1}{2}\right)^3$

73) $\left(\frac{1}{2}\right)^5$

75) $\left(\frac{7}{12}\right)^3$

77) $\left(\frac{4}{5}\right)^2$

Evaluate the expression using a calculator. Round to two decimal places.

81) $-243^{1/5}$

83) $10^{1/2}$

87) $\sqrt[3]{28}$

89) $\sqrt[4]{9}$

Let $f(x) = 6x - 11$ and $g(x) = 4x^2$.

Perform the indicated operation and state the domain.

91) $f(x) + g(x)$

93) $f(x) \cdot g(x)$

Domain:

Domain:

95) $f(g(x))$

97) $\frac{f(x)}{g(x)}$

Domain:

Domain:

99) $f(f(x))$

Domain:

The yearly cost for residents to attend a state university has increased from \$5200 to \$9000 in the last 5 years.

- a) To the nearest tenth of a percent, what has been the average annual growth rate in cost?

Initial amount:

Final amount:

Number of years:

The average annual growth rate was about

- b) If this growth rate continues, what will the cost be in 5 more years?

In 5 more years, the cost will be

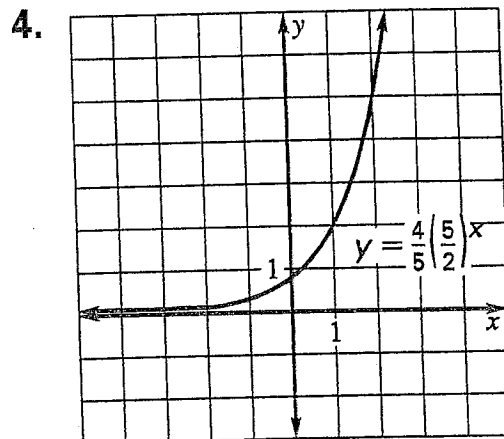
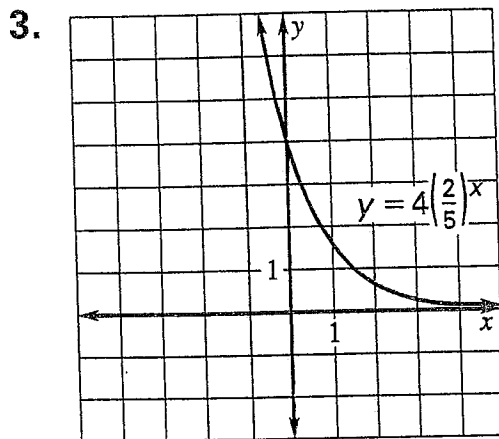
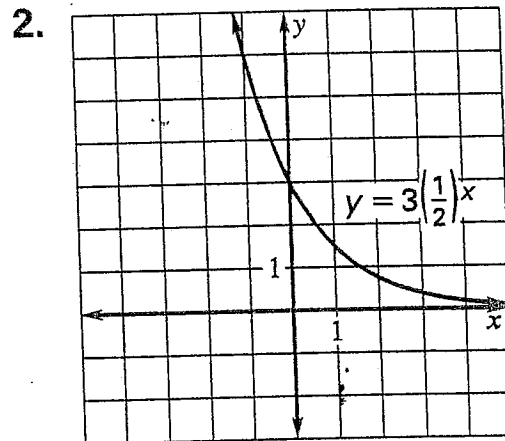
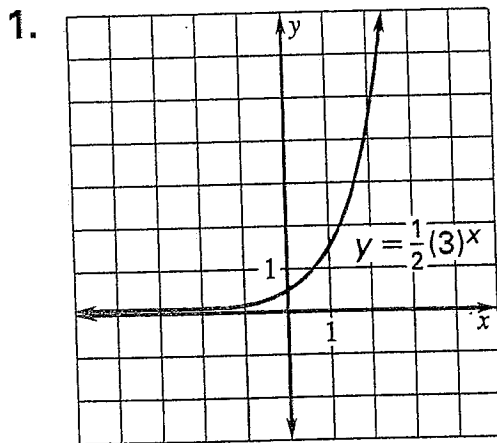
Section 7.2, Exponential Decay

Two important kinds of functions are *exponential growth functions* and *exponential decay functions*.

Exponential growth: $y = ab^x$, where $a > 0$ and $b > 1$
 For these functions, y increases as x increases and the graph approaches the x -axis as x decreases.

Exponential decay: $y = ab^x$, where $a > 0$ and $0 < b < 1$
 For these functions, y increases as x decreases and the graph approaches the x -axis as x increases.

Classify each function as an exponential growth function or an exponential decay function.



GOAL

Graph exponential decay functions and use exponential decay functions to model real-life situations

VOCABULARY

An exponential decay function has the form $f(x) = ab^x$, where $a > 0$ and $0 < b < 1$.

An exponential decay model has the form $y = a(1 - r)^t$, where y is the quantity after t years, a is the initial amount, r is the percent decrease expressed as a decimal, and the quantity $1 - r$ is called the **decay factor**.

EXAMPLE 1**Recognizing Exponential Growth and Decay**

State whether $f(x)$ is an exponential growth or exponential decay function.

a. $f(x) = 4\left(\frac{1}{3}\right)^x$

b. $f(x) = 5\left(\frac{3}{4}\right)^{-x}$

c. $f(x) = 2(0.15)^x$

SOLUTION

a. Because $b = \frac{1}{3}$, and $0 < b < 1$, f is an exponential decay function.

b. Rewrite the function without negative exponents as $f(x) = 5 \cdot \left(\frac{4}{3}\right)^x$. Because $b = \frac{4}{3}$, and $b > 1$, f is an exponential growth function.

c. Because $b = 0.15$, and $0 < b < 1$, f is an exponential decay function.

Exercises for Example 1

State whether the function represents *exponential growth* or *exponential decay*.

1. $f(x) = 3 \cdot 4^x$

2. $f(x) = 2 \cdot (0.75)^x$

3. $f(x) = 4\left(\frac{1}{3}\right)^x$

4. $f(x) = 4\left(\frac{6}{5}\right)^x$

5. $f(x) = 3\left(\frac{1}{4}\right)^{-x}$

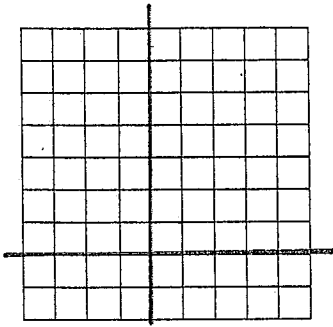
6. $f(x) = 7\left(\frac{5}{2}\right)^{-x}$

$y = ab^{x-h} + k$ start $y = ab^x$
 translate horizontally 'h'
 vertically 'k'

Graph the function. State the domain and range.

7. $y = 2\left(\frac{1}{4}\right)^x$

x	y
0	
1	

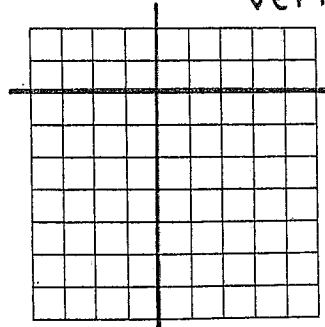


D:

R:

8. $y = -3\left(\frac{1}{2}\right)^x$

x	y
0	
1	

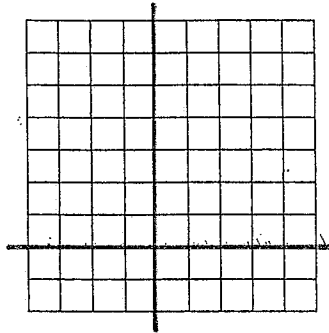


D:

R:

9. $y = 4\left(\frac{3}{4}\right)^x$

x	y
0	
1	

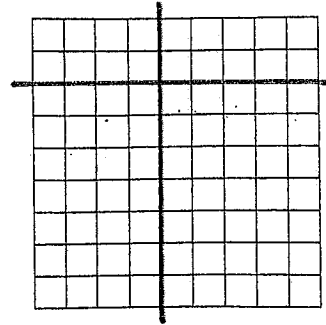


D:

R:

10. $y = -5\left(\frac{2}{3}\right)^x$

x	y
0	
1	

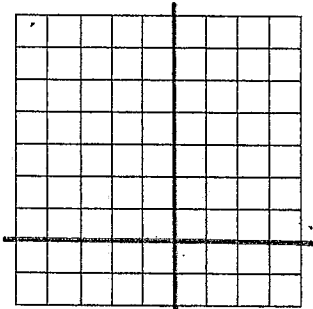


D:

R:

11. $y = 2\left(\frac{1}{2}\right)^{x+3}$

x	y
0	
1	

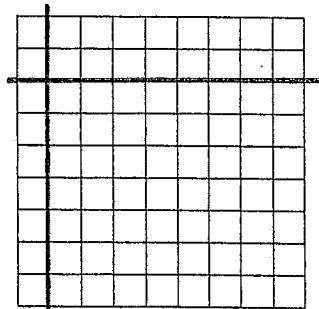


D:

R:

12. $y = -3\left(\frac{2}{3}\right)^{x-4}$

x	y
0	
1	



D:

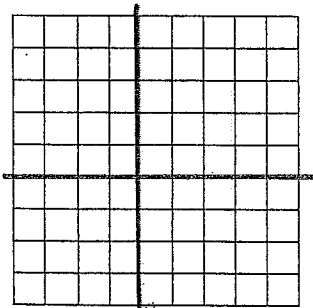
R:

shift

shift

13. $y = -\left(\frac{1}{4}\right)^x + 2$

x	y
0	
1	

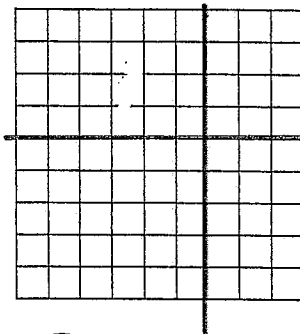


D:

R:

14. $y = 4\left(\frac{1}{2}\right)^{x+4} - 3$

x	y
0	
1	



D:

R:

shift

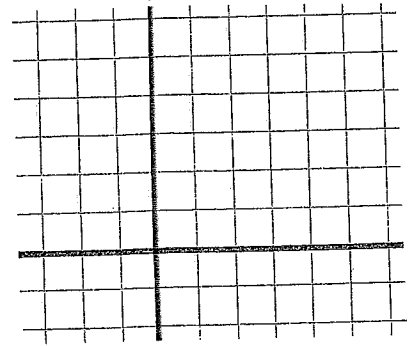
shift

shift

Name _____

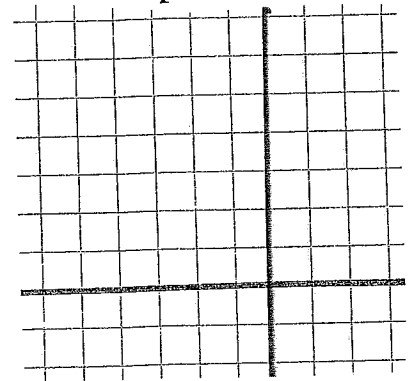
7-2 Advanced Algebra
Exponential Growth Function

Graph



Exponential Decay Function

Graph



Decay Model

quantity after t years

initial amount

percent decrease (decimal)

decay factor

- 15) An adult takes 400 milligrams of ibuprofen. Each hour h , the amount i of ibuprofen in the person's system decreases by about 29%. Write an exponential decay model.

- 16) You buy a new car for \$22,000. The value of the car decreases by 12.5% each year. Write an exponential decay model for the value of the car and estimate the value after 3 years.

NAME: _____

Table Exploration – Double Those Wheels

x	y	Growth Factor
0	1	
1	2	
2	4	
3	8	
4	16	
5	32	
6	64	

The x value is the _____

In the context of the story, the x value represents _____

The y value is the _____

In the context of the story, the y value represents _____

Is the x variable increasing at a constant amount? Yes or No

Is the growth in y multiplicative or additive? _____

What does this information tell us? _____

Compare linear functions to exponential functions. _____

NAME: Double Those Wheels

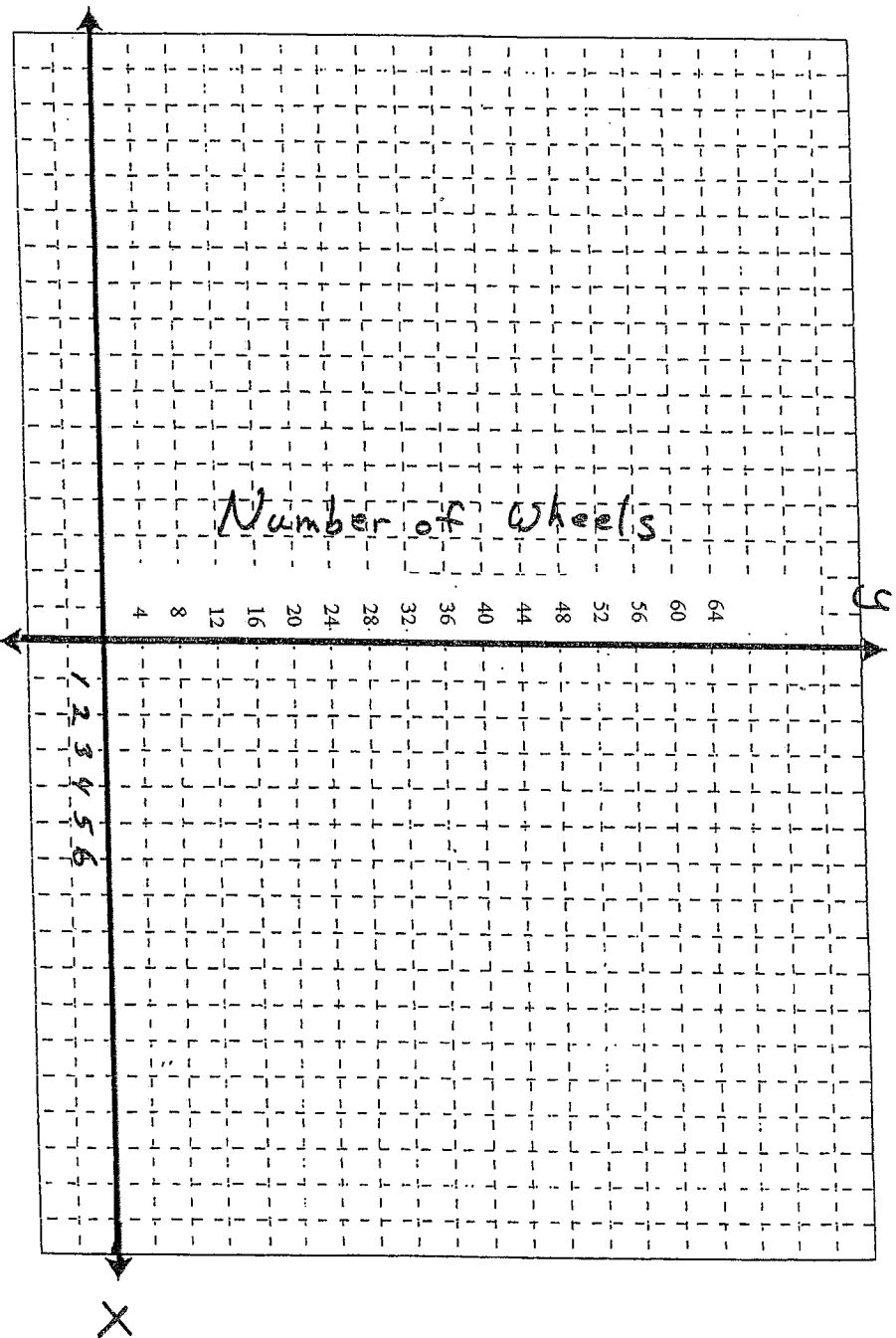
GRAPH

Function: _____

Double Those Wheels

x	y
0	
1	
2	
3	
4	
5	
6	

The growth in y
is
so the function
is



NAME: _____

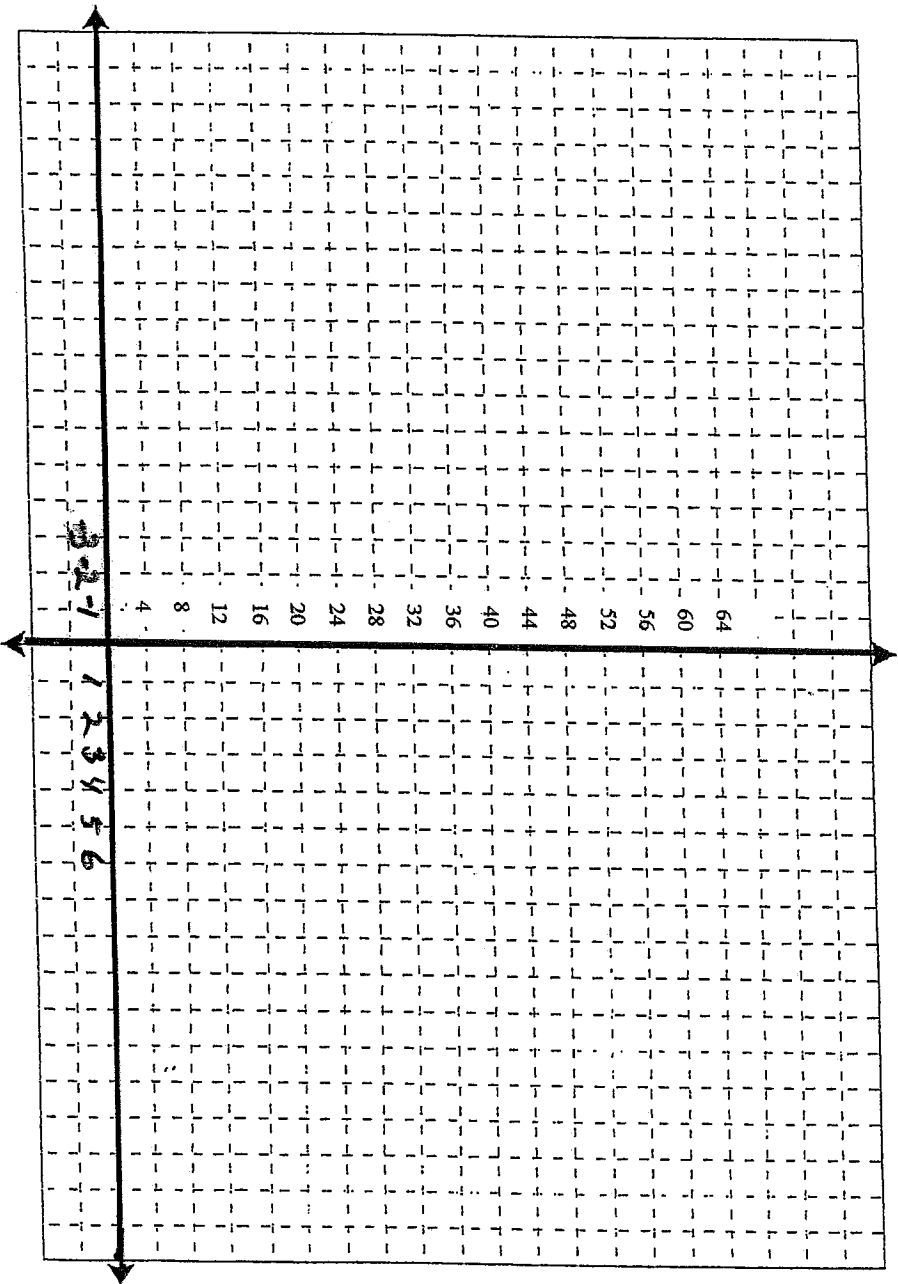
GRAPH

Function: $y = 2^x$

TABLE

times
wheels # of
Doubled wheels

x	y
-3	=
-2	=
-1	=
0	=
1	=
2	=
3	=
4	=
5	=
6	=



7-1 & 7-2 Application Problems

Applications 7-1 Exponential Growth

- 1) You deposit \$2000 in an account that pays 2.75% interest compounded quarterly. Find the balance after 3 years.

Account Balance You deposit \$3500 in an account that earns 2.5% annual interest. Find the balance after one year if the interest is compounded with the given frequency.

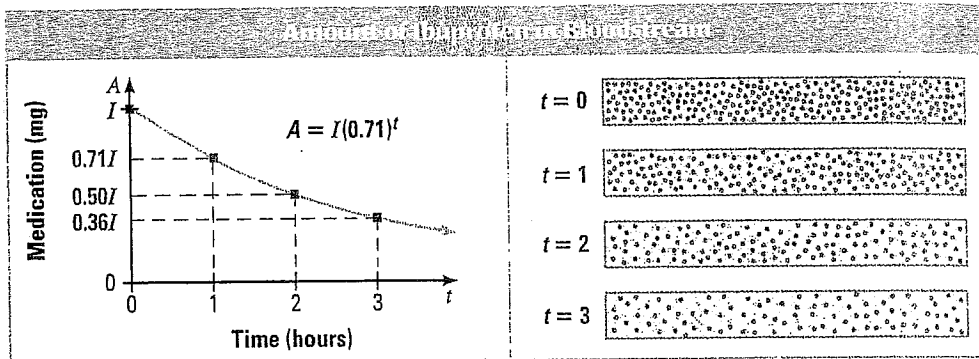
- 2) annually 3) quarterly 4) monthly

Population From 1990 to 2000, the population of California can be modeled by $P = 29,816,591(1.0128)^t$ where t is the number of years since 1990.

- 5) What was the population in 1990?
- 6) What is the growth factor and annual percent increase?
- 7) Estimate the population in 2007.

Applications 7-2 Exponential Decay

- 8) **MEDICINE** When a person takes a dosage of I milligrams of ibuprofen, the amount A (in milligrams) of medication remaining in the person's bloodstream after t hours can be modeled by the equation $A = I(0.71)^t$.



Find the amount of ibuprofen remaining in a person's bloodstream for the given dosage and elapsed time since the medication was taken.

- a. Dosage: 200 mg b. Dosage: 325 mg c. Dosage: 400 mg
 Time: 1.5 hours Time: 3.5 hours Time: 5 hours

- 9) **Depreciation** A new all-terrain vehicle (ATV) costs \$800. The value of the ATV decreases by 10% each year. Write an exponential decay model for the value of the ATV y (in dollars) after t years. Estimate the value after 5 years.

- 10) **Depreciation** You buy a new computer and accessories for \$1200. The value of the computer decreases by 30% each year. Write an exponential decay model giving the computer's value V (in dollars) after t years. What is the value of the computer after four years?

- 11) **Stereo System** You buy a new stereo system for \$640. The value of the stereo system decreases by 7% each year. Write an exponential decay model giving the stereo system's value y (in dollars) after t years. Estimate the value after five years.

Section 7.3, The Number e

Suppose you live in a country where the rate of inflation is so great that savings accounts are offered at an interest rate of 100% per year, compounded n times per year, where n is allowed to vary.

Suppose you invest \$1,000.00 in a savings account.

- Use the compound interest formula, $A = P\left(1 + \frac{r}{n}\right)^{nt}$, to find the amount of money in the account after 1 year. Your answer should be an equation showing A as a function of n .
- Complete the table. (Use a calculator.)

<i>Compounding</i>	n	<i>Amount after 1 year (dollars)</i>
Annually	1	
Quarterly	4	
Monthly	12	
Daily	365	
Hourly	8760	
Every minute	525,600	

- As n increases, the situation approaches what banks call *continuous compounding*. Try several larger values of n to guess how much money would be in the account after 1 year under continuous compounding.

7-3 Advanced Algebra

Leonhard Euler discovered the natural base e in the 1700s.
Use a calculator to complete the table.

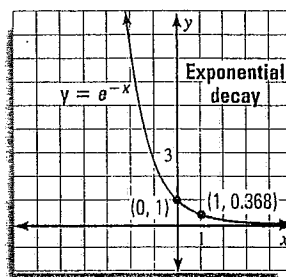
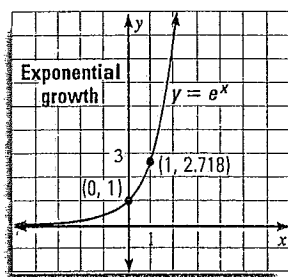
n	10^1	10^2	10^3	10^4	10^5	10^6
$(1 + \frac{1}{n})^n$	2.594					

The Natural Base e

The natural base e is irrational. It is defined as follows:

As n approaches $+\infty$, _____ approaches _____

Natural base exponential function



If $a > 0$ and $r > 0$, then _____

If $a > 0$ and $r < 0$, then _____

Simplify the expression.

15) $(100e^{0.5x})^{-2}$ 16) $\frac{e^x}{2e}$

17) $\sqrt[3]{27e^{6x}}$

18) $\frac{6e^{3x}}{4e}$

19) $e^x \cdot 4e^{2x+1}$

20) $\sqrt{4e^{2x}}$

Use a calculator to evaluate. Round to three decimal places.

21) e^8

22) $-4e^{-3}$

23) $225e^{-50}$

GOALUse the number e as the base of exponential functions**VOCABULARY**The natural base e is irrational. It is defined as follows:As n approaches $+\infty$, $\left(1 + \frac{1}{n}\right)^n$ approaches $e \approx 2.718281828459$.**EXAMPLE 1****Simplifying Natural Base Expressions**

Simplify the expression.

a. $2e \cdot e^{-4}$

b. $\frac{6e^{5x}}{2e^{3x}}$

c. $(-5e^2)^3$

SOLUTION

$$\begin{aligned} \text{a. } 2e \cdot e^{-4} &= 2e^{1+(-4)} \\ &= 2e^{-3} \\ &= \frac{2}{e^3} \end{aligned}$$

$$\begin{aligned} \text{b. } \frac{6e^{5x}}{2e^{3x}} &= 3e^{5x-3x} \\ &= 3e^{2x} \end{aligned}$$

$$\begin{aligned} \text{c. } (-5e^2)^3 &= (-5)^3 e^{(2)(3)} \\ &= -125e^6 \end{aligned}$$

Exercises for Example 1

Simplify the expression.

1. $e^{-2} \cdot e^6$

2. $5e^3 \cdot 4e^2$

3. $e^{2x} \cdot e^{4x}$

4. $(2e^3)^3$

5. $\frac{e^5}{e^2}$

6. $\frac{10e^2}{2e^4}$

Use a calculator to evaluate the expression. Round the result to three decimal places.

7. e^4

8. $e^{1/3}$

9. $e^{1.2}$

10. $2e^{-1/5}$

EXAMPLE 3

Graphing Natural Base Functions

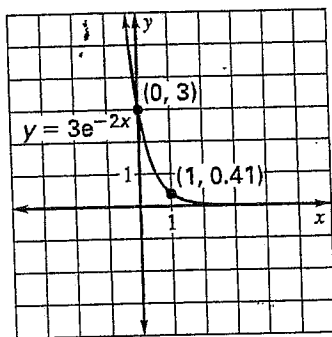
Graph the function. State the domain and range.

a. $y = 3e^{-2x}$

b. $y = \frac{1}{2}e^x - 5$

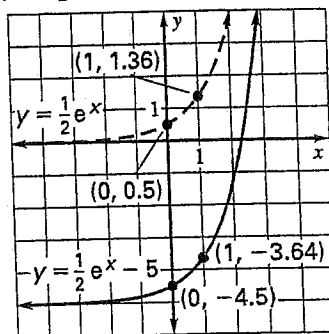
SOLUTION

a. Because $a = 3$ is positive and $r = -2$ is negative, the function is an exponential decay function. Plot points $(0, 3)$ and $(1, 0.41)$ and draw the curve.



The domain is all real numbers, and the range is $y > 0$.

b. Because $a = \frac{1}{2}$ is positive and $r = 1$ is positive, the function is an exponential growth function. Translate the graph of $y = \frac{1}{2}e^x$ down 5 units.



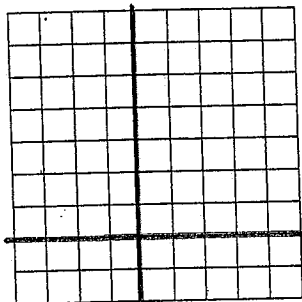
The domain is all real numbers, and the range is $y > -5$.

$y = a e^{rx}$
 $a > 0 \quad r > 0$ growth
 $a > 0 \quad r < 0$ decay

Exercises for Example 3

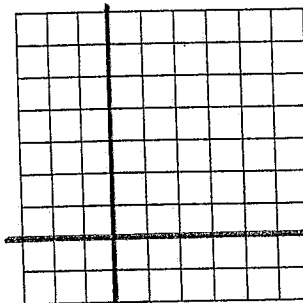
Graph the function. State the domain and range.

11. $y = 2e^{-x}$



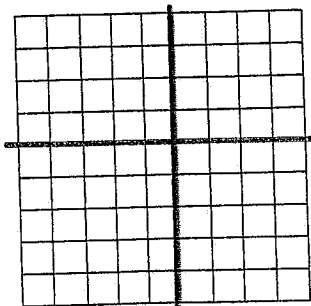
X | Y
 0 |
 1 |
 D:
 R:

12. $y = e^{x-3}$



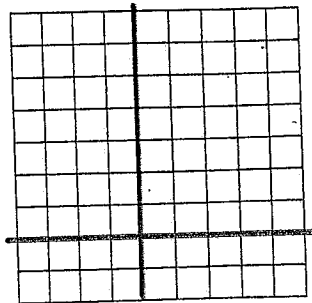
X | Y
 0 |
 1 |
 D:
 R:

13. $y = 4e^x - 3$



X | Y
 0 |
 1 |
 D:
 R:

14. $y = e^{-2x} + 1$



X | Y
 0 |
 1 |
 D:
 R:

$$f(x) = ae^{rx}$$

$r > 0$ growth
 $r < 0$ decay

Tell whether the function is an example of *exponential growth* or *decay*.

24) $f(x) = 5e^{-3x}$ 25) $f(x) = \frac{1}{4}e^{2x}$ 26) $f(x) = e^{3x}$ 27) $f(x) = e^{-9x}$

Find an equation for the inverse of the function.

28) $f(x) = 6x + 7$

29) $f(x) = \frac{1}{2}x - 10$

Solve the equation.

30) $\sqrt[3]{5x-4} + 7 = 10$

31) $\sqrt{x^2-4} = x-2$

EXAMPLE 4 *Finding the Balance in an Account*

- 32) You deposit \$1000 in an account that pays 8% annual interest compounded continuously. What is the balance after 1 year?

$P =$

$r =$

$t =$

LESSON
7.3**Practice A**

For use with pages 492–498

Simplify the expression.

1. $e^4 \cdot e^5$

2. $e^6 \cdot e^{-3}$

3. $(e^3)^2$

4. $\frac{e^{10}}{e^7}$

5. $\frac{6e^{3x}}{2e^x}$

6. $\frac{(2e)^2}{2e^2}$

Use a calculator to evaluate the expression. Round the result to three decimal places.

7. e^3

8. e^{-2}

9. e^6

10. e^0

11. e^{-4}

12. $e^{2/3}$

13. $e^{-4/3}$

14. $e^{3.1}$

Tell whether the function is an example of *exponential growth* or *exponential decay*.

15. $f(x) = e^x$

16. $f(x) = e^{-x}$

17. $f(x) = 3e^x$

18. $f(x) = \frac{1}{3}e^{2x}$

19. $f(x) = 2e^{-2x}$

20. $f(x) = e^{-x/2}$

Algebra II 7-3

27. **Finance** You deposit \$1500 in an account that pays 3.25% annual interest compounded continuously. What is the balance after five years?

28. **Population** The population of a city can be modeled by $P = 125,000e^{0.02t}$ where t is the number of years since 1990. What was the population in 1995?

Finance You deposit \$2200 in an account that pays 3% annual interest. After 15 years, you withdraw the money.

a) What is the balance if the interest is compounded quarterly?

b) What is the balance if the interest is compounded continuously?

Exponential Functions

$$y = ab^x$$

$$a > 0$$

$$0 < b < 1$$

$$b > 1$$

$$y = ab^{x-h} + k \quad \begin{matrix} h: \\ k: \end{matrix}$$

ex $y = ab^{x+3} + 5 \quad \begin{matrix} 3 \\ 5 \end{matrix}$

ex $y = ab^{x-2} - 7 \quad \begin{matrix} 2 \\ 7 \end{matrix}$

Tell whether the function represents *exponential growth* or *exponential decay*.

1. $f(x) = \frac{1}{2}\left(\frac{5}{7}\right)^x$

2. $f(x) = \frac{1}{3}\left(\frac{7}{5}\right)^x$

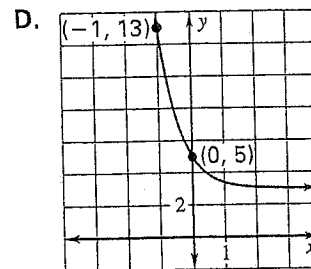
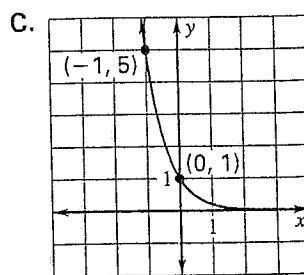
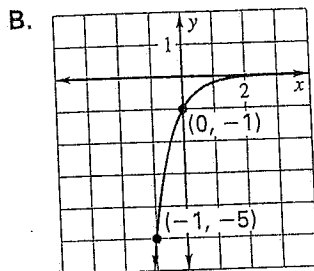
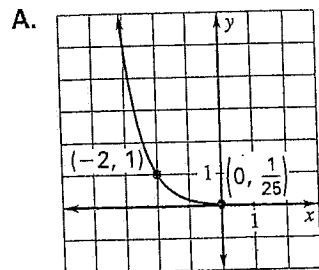
Match the function with its graph.

3. $y = \left(\frac{1}{5}\right)^x$

4. $y = \left(\frac{1}{5}\right)^{x+2}$

5. $y = 2\left(\frac{1}{5}\right)^x + 3$

6. $y = -\left(\frac{1}{5}\right)^x$



Simplify the expression.

7. $4e^{-3} \cdot e^5$

8. $(-2e^{2x})^2$

9. $\frac{5e^x}{6e}$

10. $\frac{12e^x}{e^{4x}}$

11. $\sqrt[3]{27e^{6x}}$

Interest formulas

Continuous

12. You deposit \$5,000 in an account that pays 7% annual interest compounded continuously. What is the balance after 2 years?

$$f(x) = ae^{rx}$$

$$a > 0$$

$$r > 0$$

$$r < 0$$

Rounding: 1/2 person?

Graph the function on your calculator. State the domain and range.

13) $y = 4^x - 1$

14) $y = 3^{x+1} + 2$

15) $y = \frac{1}{2} \cdot 5^{x-1}$

Domain:

Domain:

Domain:

Range:

Range:

Range:

16) $y = -2\left(\frac{1}{6}\right)^x$

17) $y = \left(\frac{5}{8}\right)^x + 2$

18) $y = -2 \cdot 6^{x-3} + 3$

Domain:

Domain:

Domain:

Range:

Range:

Range:

Simplify the expression.

19) $2e^3 \cdot e^4$

20) $4e^{-5} \cdot e^7$

21) $(-3e^{2x})^2$

22) $(5e^{-3})^{-4x}$

23) $\frac{3e^x}{4e}$

24) $\frac{6e^x}{e^{5x}}$

25) $\sqrt{16e^{2x}}$

26) $\sqrt[3]{125e^{6x}}$

27) Radioactive Decay: One hundred grams of radium is stored in a container.

The amount R (in grams) of radium present after t years can be modeled by

$$R = 100e^{-0.00043t}$$

How much of the radium is present after 10,000 years?

You deposit \$1600 in a bank account. Find the balance after 3 years for:

28) 2.5% annual interest compounded monthly

29) 1.75% annual interest compounded quarterly

30) 4% annual interest compounded yearly

You want to have \$2500 after 2 years. Find the amount you should deposit for each of the situations described below.

31) The account pays 2.25% annual interest compounded monthly

32) The account pays 2% annual interest compounded quarterly

33) The account pays 5% annual interest compounded yearly