Date

Hour

Section 8.1 Model Inverse and Joint Variation

GOAL

Write and use inverse variation models and joint variation models

VOCABULARY

Inverse variation is the relationship of two variables x and y if there is a nonzero number k such that xy = k, or $y = \frac{k}{x}$.

The nonzero constant k is called the **constant of variation**.

Joint variation occurs when a quantity varies directly as the product of two or more other quantities. For instance, if z = kxy where $k \neq 0$, then z varies jointly with x and y.

Recall: Direct Variation

v = kx

Tell whether x and y show direct variation, inverse variation, or neither.

1.
$$xy = 8$$

2.
$$y = x + 5$$

3.
$$y = \frac{x}{2}$$

4.
$$x = \frac{y}{3}$$

The variables x and y vary inversely. Use the given values to write an equation relating x and y. Then find y when x = 4.

5.
$$x = 10, y = 2$$

6.
$$x = -3, y = 3$$

7.
$$x = 2, y = 8$$

The variable z varies jointly with x and y. Use the given values to find an equation that relates the variables. Then find z when x = 2 and y = 8.

8.
$$x = 4$$
, $y = 3$, $z = 24$

9.
$$x = 8, y = -54, z = 144$$

10.
$$x = 1, y = \frac{1}{8}, z = 4$$

Write an equation for the given relationship.

RELATIONSHIP

EQUATION

- **a.** y varies directly with x.
- b. y varies inversely with x.
- **c.** z varies jointly with x and y.
- d. y varies inversely with the square of x.
- **e.** z varies directly with y and inversely with x.
- Vith the length l of the handle. Write an equation relating F and l, given that 250 pounds of force must be exerted to loosen a bolt when using a wrench with a handle 6 inches long. How much force must be exerted when using a wrench with a handle 24 inches long?

Write an equation for the relationship.

Relationship

Example

Equation

- a. m varies jointly with n, p, and q.
- m = ____

b. *r* varies inversely with *s*.

- *r* =
- **c.** *x* varies inversely with the cube of *y*.
- **(=** ___
- **d.** p varies jointly with x and y and inversely with m.
- *p* = ____
- e. t varies directly with u and inversely with w.
- t =

Checkpoint Complete the following exercises.

14. The variable z varies jointly with x and y. Also, z = -44 when x = 4 and y = -1. Write an equation that relates x, y, and z. Find z when x = 6 and y = 3.

15. Write an equation for the relationship: x varies jointly with y and z and inversely with the square of t.

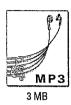
Write an inverse variation model

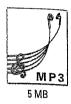
MP3 PLAYERS The number of songs that can be stored on an MP3 player varies inversely with the average size of a song. A certain MP3 player can store 2500 songs when the average size of a song is 4 megabytes (MB).

- Write a model that gives the number n of songs that will fit on the MP3 player as a function of the average song size s (in megabytes).
- Make a table showing the number of songs that will fit on the MP3 player if the average size of a song is 2 MB, 2.5 MB, 3 MB, and 5 MB as shown below. What happens to the number of songs as the average song size increases?









Solution

STEP 1 Write an inverse variation model.

Write general equation for inverse variation.

Substitute 2500 for n and 4 for s.

Solve for

A model is

STEP 2 Make a table of values.

Average size of song (MB), s

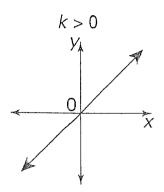
Number of songs, n

2	2.5	3	5
A35.A21146			
S.D. LECTOR STATE OF THE STATE			
The second secon	AND THE PROPERTY OF THE PARTY O	Property of the Comment of the Comme	

From the table, you can see that the number of songs that will fit on the MP3 player decreases as the average song size increases.

Graphs of Direct-variation formulas

y = kxy varies directly as x.



k < 0

y

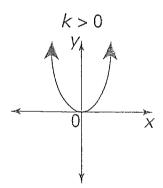
0

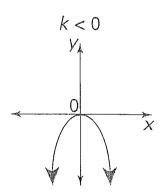
x

line

D = R = the set of all real numbers

 $y = kx^2$ y varies directly as the square of x.





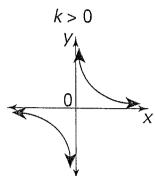
parabola

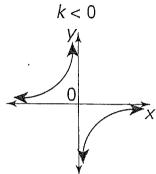
$$R = \{y: y \ge 0\}$$

$$D$$
 = the set of all real numbers $R = \{y: y \le 0\}$

Graphs of Inverse-variation formulas

 $y = \frac{k}{X}$ y varies inversely as x.



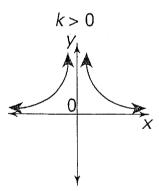


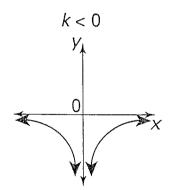
hyperbola

D = R = the set of all nonzero real numbers

$$y = \frac{k}{x^2}$$

y varies inversely as the square of x.





inverse-square curve

D =the set of all nonzero real numbers

$$R = \{y: y > 0\}$$

D =the set of all nonzero real numbers

$$R = \{y: y < 0\}$$

Date

Hour_

Multiply and Divide Rational Expressions 8-4

VOCABULARY

Simplified form of a rational expression A rational expression in which its numerator and denominator have no common factors (other than ± 1)

SIMPLIFYING RATIONAL EXPRESSIONS

Let a, b, and c be nonzero real numbers or variable expressions. Then the following property applies.

$$\frac{a\phi}{b\phi} =$$

Divide out common factor c.

Simplify a rational expression

$$\frac{x^2 + 7x + 10}{x^2 - 4} = \frac{}{}$$
 Factor numerator and denominator.

and denominator.

Divide out common factor.

Simplified form.

Exercises

If possible, simplify the rational expression.

1.
$$\frac{y^2 - 81}{2y - 18}$$

2.
$$\frac{2x-3}{4x-6}$$

3.
$$\frac{x+3}{x^2+6x+9}$$

4.
$$\frac{y^2 - 7y}{y^2 - 8y + 7}$$

1.
$$\frac{y^2 - 81}{2y - 18}$$
 2. $\frac{2x - 3}{4x - 6}$ 3. $\frac{x + 3}{x^2 + 6x + 9}$ 4. $\frac{y^2 - 7y}{y^2 - 8y + 7}$ 5. $\frac{x^2 + 5x + 6}{x^3 + 3x^2}$

multiplying rational expressions

Let a, b, c, and d be nonzero real numbers or variable expressions. The rule for multiplying rational expressions is the same as the rule for multiplying numerical fractions: multiply _______, and write the new fraction in simplified form.

$$\frac{a}{b} \cdot \frac{c}{d} =$$
 Simplify $\frac{ac}{bd}$ if possible.

க்கூறிஜே Multiply rational expressions

$$\frac{2x^2 + 4x}{x^2 - 4x - 12} \cdot \frac{x^2 - 9x + 18}{2x}$$

Divide out common factors and write in simplified form.

ভিৰ্মাচাৰিই Multiply a rational expression by a polynomial

$$\frac{x-4}{x^3+1} \cdot (x^2-x+1)$$

$$=\frac{x-4}{x^3+1}\cdot \frac{W}{e}$$

Write polynomial as rational expression.

Divide out common factor and write in simplified form.

Multiply the rational expressions. Simplify the result.

6.
$$\frac{5x-20}{5x+15} \cdot \frac{2x+6}{x-4}$$

7.
$$\frac{12-x}{3} \cdot \frac{3}{x-12}$$

$$2 \frac{x^2 + 2x - 3}{x + 2} \cdot \frac{x^2 + 2x}{x^2 - 1}$$

9.
$$\frac{x^2-2x}{x^2+2x+1} \cdot \frac{x^2+4x+3}{x^2+3x}$$

DIVIDING RATIONAL EXPRESSIONS

To divide one rational expression by another, multiply the first expression by the reciprocal of the second

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} =$$

 $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} =$ Simplify $\frac{ad}{bc}$ if possible.

🖎 Ample 4 🎤 Divide rational expressions

$$\frac{3}{x+7} \cdot \frac{8x^2 - 8x}{x^2 + 6x - 7}$$

$$= \frac{3}{x+7} \cdot \cdot$$

$$= \frac{3}{x+7} \cdot \cdot$$

$$=\frac{3}{x+7}$$

Multiply by reciprocal.

$$=\frac{3}{x+7}$$

Factor.

Divide out common factors and write in simplified form.

Exercises

Divide the rational expressions. Simplify the result.

10.
$$\frac{48x^2}{y} \div \frac{36xy^2}{5}$$

$$11. \qquad \frac{x^2}{x^2-1} \div \frac{3x}{x+1}$$

12.
$$\frac{2x^3 - 12x^2}{x^2 - 4x - 12} \div \frac{8x^3 + 24x^2}{x^2 + 9x + 18}$$

1. Simplify the expression.

$$\frac{x^2 - 2x - 15}{x^2 + 4x + 3}$$

2. Multiply the expression.

$$\frac{6x^2 + 18x}{x^2 + x - 6} \cdot \frac{x^2 - x - 2}{x^2 - 7x - 8}$$

3.
$$\frac{-2x^2}{x^3-27} \cdot (x^2+3x+9)$$

4.
$$\frac{x-5}{9x^2-18x} \div \frac{2x^2-11x+5}{2x^2-5x+2}$$

13. Factor
$$x^3 + 5x^2 - x - 5$$

Perform the indicated operations. Simplify the result.

$$\frac{x^2 - x - 12}{8x^2} \div \frac{x^3 + 3x^2}{8x^3 - 2x^2} \div \frac{4x - 1}{x + 2}$$
15.
$$\frac{2x^2 + x - 15}{2x^2 - 11x - 21} \cdot (6x + 9) \div \frac{2x - 5}{3x - 21}$$

Find the product.

16.
$$(x+7)(x-1)$$
17. $(x+3)(x^2+3x+2)$
18. $x(x^2-4)(5-6x^3)$

A	lgebra	II

Name	

Hour Date

Add and Subtract Rational Expressions 8-5

VOCABULARY .

Complex fraction A fraction that contains a fraction in its numerator or denominator

add (Subtract) with like denominators

To add (or subtract) rational expressions with like denominators, simply add (or subtract) their . Then place the result over the common denominator. Let a, b, and c be polynomials with $c \neq 0$.

Addition
$$\frac{a}{c} + \frac{b}{c} =$$

Subtraction
$$\frac{a}{c} - \frac{b}{c} =$$

Add with like denominators Example 1

$$\frac{9}{6x} + \frac{2}{6x} = \frac{2}{6x} = \frac{3}{6x}$$

Add numerators.

ADD (SUBTRACT) WITH UNLIKE DENOMINATORS

To add (or subtract) two rational expressions with unlike denominators, find the least common (LCD), which is the least common (LCM) of the denominators.

Rewrite each rational expression using the LCD, then add (or subtract) using the procedure for like denominators. Let a, b, c, and d be polynomials with $c \neq 0$ and $d \neq 0$.

Addition
$$\frac{a}{c} + \frac{b}{d} = \frac{ad}{cd} + \frac{bc}{cd} =$$

Subtraction
$$\frac{a}{c} - \frac{b}{d} = \frac{ad}{cd} - \frac{bc}{cd} =$$

Add:
$$\frac{5}{4x^2} + \frac{x+1}{2x^2+4x}$$

To find the LCD, factor each denominator and write the highest power to which each factor occurs. Note that $4x^2 =$ ____ and $2x^2 + 4x =$ ____, so the LCD is ___ = ___

$$\frac{5}{4x^2} + \frac{x+1}{2x^2 + 4x}$$

$$= \frac{5}{4x^2} + \frac{x+1}{-}$$

$$= \frac{5}{4x^2} \cdot \frac{1}{-} + \frac{x+1}{-}$$

$$=\frac{5}{4x^2}\circ \frac{1}{x} - \frac{x+1}{x} + \frac{x+1}{x}$$

医comple Subtract with unlike denominators

Subtract: $\frac{7}{3x-9} - \frac{x+4}{x^2-9}$ $\frac{7}{3x-9} - \frac{x+4}{x^2-9}$ $\frac{7}{x^2-9} - \frac{x+4}{x^2-9}$

$$\frac{7}{3x-9} - \frac{x+4}{x^2-9}$$

$$=$$
 $\frac{7}{5}$ $\frac{x+4}{5}$

$$=$$
 $\frac{7}{x+4}$

Adding with Unlike Denominators

Perform the indicated operation and simplify.

1.
$$\frac{1}{2} + \frac{3}{x^2}$$

$$2. \ \frac{3}{2x} + \frac{x}{2x^2 + 6x}$$

3.
$$\frac{3}{x+5} + \frac{4}{x+1}$$

4.
$$\frac{x}{x-1} + \frac{3x}{x^2-1}$$

Subtracting with Unlike Denominators

Perform the indicated operation and simplify.

$$5. \ \frac{2x}{x+2} - \frac{8}{x^2+2x}$$

6.
$$\frac{5x}{x^2-4}-\frac{7}{x-2}$$

7.
$$\frac{3x+1}{x^2} - \frac{x-2}{x^3}$$

8.
$$\frac{x}{x+3} - \frac{6}{x+2}$$

SIMPLIFYING COMPLEX FRACTIONS

A complex fraction is a fraction that contains a fraction in its numerator or denominator.

Method 1: If necessary, simplify the numerator and denominator by _writing each as a _____. Then divide the numerator by the denominator.

Method 2: Multiply the numerator and the denominator by the _____ of every fraction in the numerator and denominator. Then simplify.

ப்பாற் (Method 1)

Simplify:
$$\frac{\frac{8}{5x}}{\frac{x+1}{15} + \frac{4}{15}}$$

$$\frac{\frac{8}{5x}}{\frac{x+1}{15} + \frac{4}{15}} = \frac{\frac{8}{5x}}{}$$
Write denominator as a single fraction.
$$= \frac{8}{5x} \cdot \qquad \text{Divide numerator by denominator.}$$

$$= \qquad \qquad \text{Simplify.}$$

Simplify a complex fraction (Method 2)

Simplify: $\frac{\frac{2x}{x+1}}{\frac{1}{3x} + \frac{2}{x+1}}$

The LCD of all the fractions in the numerator and denominator is

$$\frac{\frac{2x}{x+1}}{\frac{1}{3x} + \frac{2}{x+1}} = \frac{\frac{2x}{x+1}}{\frac{1}{3x} + \frac{2}{x+1}}$$

= Simplify.

Multiply numerator and denominator

by the LCD.

Simplify.

Simplifying a Complex Fraction

9.
$$\frac{\frac{x^2}{x^2 - 1}}{\frac{3x}{x + 1}}$$

10.
$$\frac{2-\frac{1}{x}}{x}$$

11.
$$\frac{1+\frac{1}{x}}{1-\frac{1}{x}}$$

$$\frac{\frac{6}{x-1}-3}{\frac{3}{x}}$$

Checkpoint Perform the indicated operation and simplify.

1.
$$\frac{8x}{3+x} - \frac{7}{3+x}$$

2.
$$\frac{13}{3x} + \frac{2}{3x}$$

$$3. \frac{x+1}{x^2+6x+9} + \frac{6}{x^2-9}$$

$$4. \frac{8}{4x^2} - \frac{2+x}{8x^2 - 12x}$$

Simplify the complex fraction.

$$5. \frac{\frac{6x}{4} - \frac{x}{4}}{\frac{8}{2} + \frac{3x}{2}}$$

$$6. \ \frac{\frac{5}{2x} + \frac{7}{x^2}}{\frac{3}{x^2} - \frac{1}{2x}}$$

Solve the equation.

(a)
$$\frac{3}{4}x + \frac{1}{2} = x - \frac{5}{6}$$
 (b) $-\frac{1}{12}x - 3 = \frac{5}{2}$ (c) $x = 12 + \frac{5}{6}x$

$$(-\frac{1}{12}x - 3) = \frac{5}{2}$$

$$x = 12 + \frac{5}{6}$$

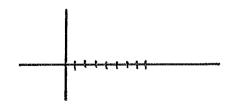
Solve the quadratic equation.

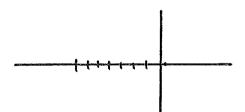
$$5x^2 - 8 = 4(x^2 + 3)$$
 $3(x - 5)^2 = 27$ $2x(x + 6) = 7 - x$

$$3(x-5)^2 = 2^x$$

$$2x(x+6) = 7 - x$$







Find the least common denominator.

9)
$$\frac{1}{x^2-3x-28}$$
, $\frac{x}{x^2+6x+8}$

$$\frac{20}{x+1}$$
 $\frac{1}{4} - \frac{7}{x+1}$

$$\frac{\frac{1}{4x+3} - \frac{5}{3(4x+3)}}{\frac{x}{4x+3}}$$

Algebra II

Name _____

Date

_____ Hour ____

8-5 Practice B

Find the least common denominator.

7.
$$\frac{2}{x-3}$$
, $\frac{3}{2x+3}$

2.
$$\frac{8}{x+2}, \frac{2x}{x-1}$$

3.
$$\frac{3x}{x-2}$$
, $\frac{2}{x^2-4}$

4.
$$\frac{x}{3x(x+3)}$$
, $\frac{1}{x^2-9}$, $\frac{4}{x(x-3)}$

Perform the indicated operation and simplify.

$$5. \ \, \frac{2}{3x+1} + \frac{x}{3x+1}$$

6.
$$\frac{x}{x^2-4x+3}+\frac{5}{x-3}$$

7.
$$\frac{3x}{x-5} - \frac{2}{x^2-25}$$

8.
$$\frac{3}{x} + \frac{2}{x-2} - \frac{2}{x^2}$$

$$9. \ \frac{x}{x+3} - \frac{3}{x+2} - \frac{1}{x^2 + 5x + 6}$$

10.
$$\frac{2x}{x^2+4x+4}+\frac{x-1}{x(x+2)}$$

11.
$$2 + \frac{x}{x^2 - 2}$$

12.
$$\frac{x-2}{x^2+x-12} + \frac{x}{x^2-2x-3}$$

Simplify the complex fraction.

13.
$$\frac{\frac{2}{x} + \frac{3}{x-1}}{\frac{1}{2x-2}}$$

14.
$$\frac{\frac{3}{x+2} + \frac{2}{3}}{\frac{2x}{x+2} - \frac{1}{x}}$$

15.
$$\frac{\frac{3x}{2x-1}-2}{\frac{5}{4x}-\frac{x}{2x-1}}$$

Apply the Distance and Midpoint Formulas

Name

Goal • Find the length and midpoint of a line segment.

VOCABULARY

Distance formula

Midpoint formula

THE DISTANCE FORMULA

The distance d between (x_1, y_1) and (x_2, y_2) is $d = \sqrt{()^2 + ()^2}.$

$$d = \sqrt{()^2 + ()^2}.$$

Example Find the distance between two points

Find the distance between (-5, -3) and (3, 6).

Let
$$(x_1, y_1) = (-5, -3)$$
 and $(x_2, y_2) = (3, 6)$.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Let
$$(x_1, y_1) = (-5, -3)$$
 and $(x_2, y_2) = (3, 6)$.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

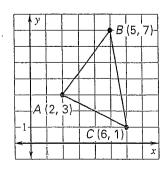
$$= \sqrt{\underline{}} = \underline{}$$

Example 2 Classify a triangle using the distance formula

Classify $\triangle ABC$ as scalene, isosceles, or equilateral.

$$AB = \bigvee$$

$$AC = \sqrt{}$$



- **1.** Find the distance between (-7, 3) and (5, -2).
- **2.** The vertices of a triangle are T(2, 1), U(4, 6), and V(7, 3). Classify $\triangle TUV$ as scalene, isosceles, or equilateral.

THE MIDPOINT FORMULA

A line segment's midpoint is _____ from the segment's endpoints. The midpoint formula describes the _____ of a line segment joining $A(x_1, y_1)$ and $B(x_2, y_2)$ as follows:

$$M\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

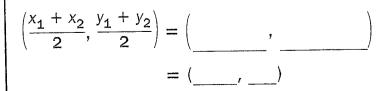
In words, each coordinate of M is the _____ of the corresponding coordinates of A and B.

Midpoint M

Example 3 Find the midpoint of a line segment

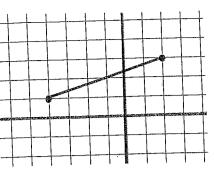
Find the midpoint of the line segment joining (-6, 5) and (2, -3).

Let
$$(x_1, y_1) = (-6, 5)$$
 and $(x_2, y_2) = (2, -3)$.



Example 4 Find a perpendicular bisector

Write an equation for the perpendicular bisector of the line segment joining A(-4, 1) and B(2, 3).



Solution

1. Find the midpoint of the line segment.

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\underline{}, \underline{}\right) = \left(\underline{}, \underline{}\right)$$

2. Calculate the slope of AB.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \underline{\qquad} = \underline{\qquad} = \underline{\qquad}$$

3. Find the slope of the perpendicular bisector.

$$-\frac{1}{m} = =$$

y = _____.

An equation for the perpendicular bisector of AB is

Checkpoint Complete the following exercises.

- 3. Find the midpoint of the line segment joining (-6, 5)and (1, 1).
- 4. Write an equation for the perpendicular bisector of the line segment joining A(-5, 6) and (3, -2).

Graph and Write Equations of Circles

Name

· Graph and write equations of circles.

· · · · · · · · · · · · · · · · · · ·

STANDARD EQUATION OF A CIRCLE WITH CENTER AT THE ORIGIN

The standard form of the equation of a circle with center at (0, 0) and radius r is as follows:

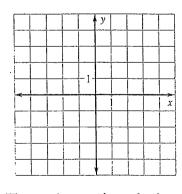
$$x^2 + y^2 =$$

Example 1	<u> </u>		, ,	•	# E
- Carried and Line Bridge	Fironn	20	anustian	At a	CIPCIO
	WI CELJII	CHI	GUUGUUI	u a	CHUIC

Graph $y^2 = -x^2 + 16$. Identify the radius of the circle.

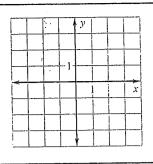
Solution

- **1. Rewrite** the equation $y^2 = -x^2 + 16$ in standard form
- 2. Identify the center and radius. From the equation, the graph is a circle centered at the origin with radius $r = \underline{} = \underline{}$.
- 3. Draw the circle. First plot several convenient points that are 4 units from the origin, such as (0,),
 - $(4, \underline{\hspace{1cm}})$, $(0, \underline{\hspace{1cm}})$, and $(-4, \underline{\hspace{1cm}})$. Then draw the circle that passes through the points.



Checkpoint Graph the equation. Identify the radius.

1.
$$x^2 = 4 - y^2$$



Example 2 Write an equation of a circle

The point (-3, 4) lies on a circle whose center is the origin. Write the standard form of the equation of the circle.

The circle's radius r must be the distance between the center and (-3, 4). Use the distance formula.

$$r = \sqrt{(\underline{})^2 + (\underline{})^2}$$

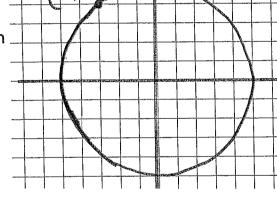
$$= \sqrt{\underline{}} = \sqrt{\underline{}} = \underline{}$$

Use the standard form with r = to write anequation of the circle.

$$x^2 + y^2 = r^2$$
 Standard form
 $x^2 + y^2 = 2$ Substitute for r.
 $x^2 + y^2 = 2$ Simplify.

$$x^2 + y^2 = 2$$

$$x^2 + y^2 =$$



Example 3 Find a tangent line

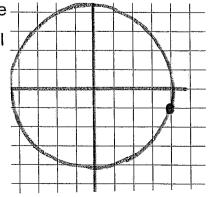
Write an equation of the line tangent to the circle $x^2 + y^2 = 17$ at (4, -1).

A line tangent to a circle and the radius to the point of tangency are perpendicular. The radius with endpoint

(4, -1) has slope m = 1= , so the slope of the tangent line at (4, -1) is the negative reciprocal

of , or ___. An equation of the tangent line is as follows:

$$y - \underline{\hspace{1cm}} = \underline{\hspace{1cm}} (x - \underline{\hspace{1cm}})$$
 Point-slope form $y = \underline{\hspace{1cm}}$ Solve for y .



Example 4

Write a circular model

Lighthouse The beam from Oak Island Lighthouse in North Carolina can be seen for up to 24 miles. You are 18 miles east and 9 miles south of the lighthouse. Can you see the lighthouse beam?

Solution

- **1. Write** an inequality for the region lit by the beam. This region is all the points that satisfy the following inequality: $x^2 + y^2 < ___2$
- 2. Substitute the coordinates (18, 9) into the inequality.

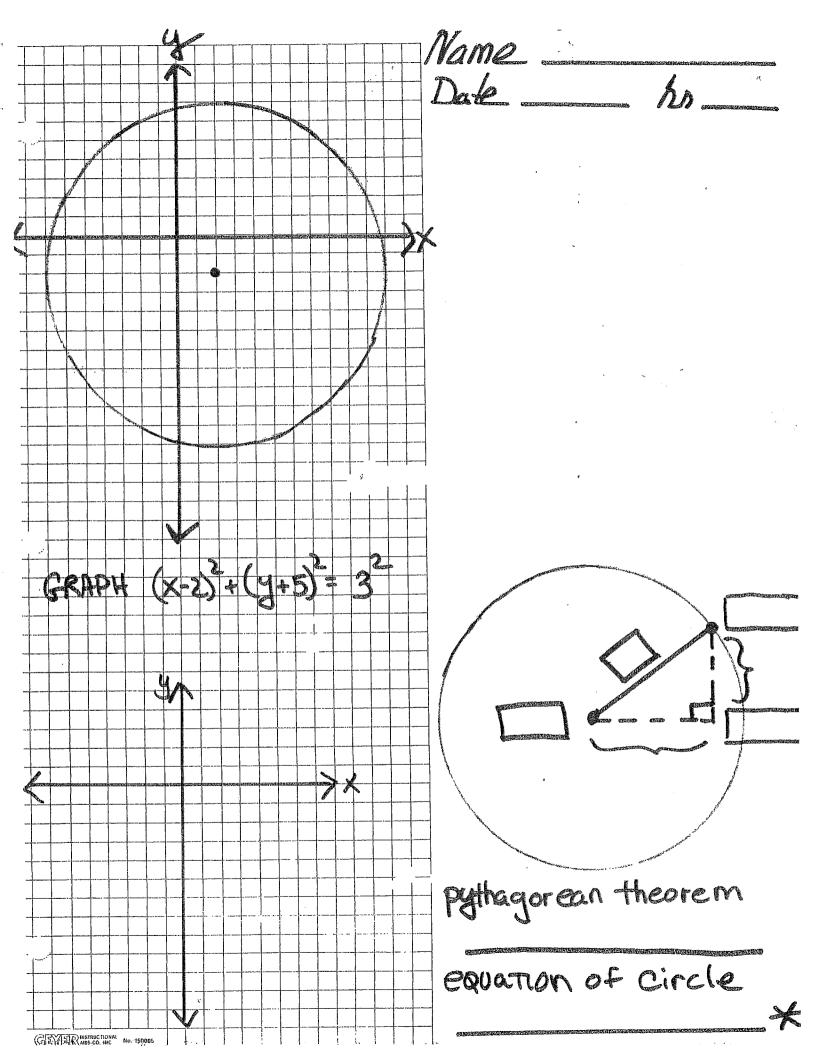
$x^2 + y^2 < $	2	Inequality
<	2	Substitute for x and y .
		The inequality is

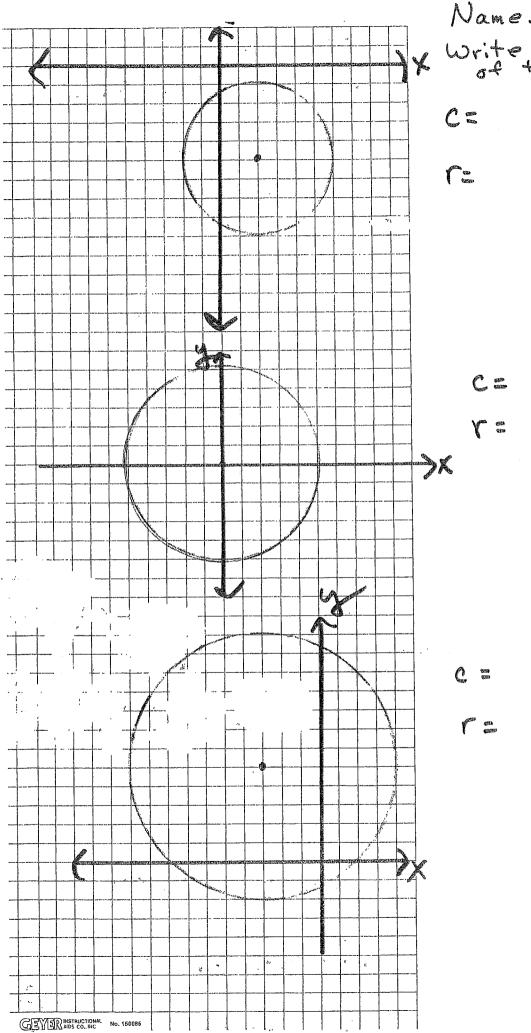
You _____ see the lighthouse beam.

Checkpoint Complete the following exercises.

- 2. Write the standard form of the equation of the circle with center at the origin that passes through the point (6, -3).
- 3. Write an equation of the line tangent to the circle $x^2 + y^2 = 34$ at (-3, -5).

4. From Example 4, suppose you are 16 miles east and 19 miles south of the lighthouse. Can you see the lighthouse beam?





Name. X Write the Equation of the circle.