Seat # ______

Algebra 2B

Name____________________
Date______________Hr________

Section 6.1, Evaluate *n*th Roots and Use Rational Exponents

GOAL **Evaluate *n*th roots and study rational exponents.**

Vocabulary

For an integer n greater than 1, if $b^n = a$, then b is an **nth root of a.**

An nth root of a is written as $\sqrt[n]{a}$ where n is the **index of the radical.**

REAL *n*th ROOTS OF *a*

Let n be an integer ($n > 1$) and let a be a real number.

If n is an even integer:

- $a < 0$ No real nth roots.
- $a = 0$ One real nth root: $\sqrt[n]{0} =$ ___
- $a > 0$ Two real nth roots: $\pm\sqrt[n]{a} =$ _______

If n is an odd integer:

- $a < 0$ One real nth root: $\sqrt[n]{a} =$ _____
- $a = 0$ One real nth root: $\sqrt[n]{0} =$ ___
- $a > 0$ One real nth root: $\sqrt[n]{a} =$ _____

Example 1 ***Find nth roots***

Find the indicated real nth root(s) of a.

a. $n = 3, a = -64$

b. $n = 6, a = 729$

Solution

a. Because $n = 3$ is odd and $a = -64$ ___ 0, -64 has __________________. Because (_____)$^3 = -64$, you can write $\sqrt[3]{-64} =$ _____ or $(-64)^{1/3} =$ _____.

b. Because $n = 6$ is even and $a = 729$ ___ 0, 729 has __________________. Because ___$^6 = 729$ and (_____)$^6 = 729$, you can write $\pm\sqrt[6]{729} =$ _____ or $\pm 729^{1/6} =$ _____.

RATIONAL EXPONENTS

Let a be a real number, and let m and n be positive integers with $n > 1$.

$a^{m/n} = (a^{1/n})^m = (\_\_\_\_\_)^m$

$a^{-m/n} = \frac{1}{a^{m/n}} = \frac{1}{(a^{1/n})^m} = \frac{1}{(\_\_\_\_\_)^m}, a \neq 0$

Example 2 *Evaluate an expression with rational exponents*

Evaluate $8^{-4/3}$

Solution

Rational Exponent Form	Radical Form
$8^{-4/3} = \frac{}{\_\_\_\_}$	$8^{-4/3} = \frac{}{\_\_\_\_}$
$= \frac{}{\_\_\_\_}$	$= \frac{}{\_\_\_\_}$
$= \frac{}{\_\_\_\_}$	$= \frac{}{\_\_\_\_}$
$= \frac{}{\_\_\_\_}$	$= \frac{}{\_\_\_\_}$

Find the indicated *n*th real root(s) of *a*.

1. $n = 2, a = 64$
2. $n = 3, a = -343$

Evaluate the expression without using a calculator.

3. $16^{-1/4}$
4. $36^{3/2}$
5. $8^{4/3}$
6. $64^{-1/6}$

Evaluate the expression using a calculator. Round the result to two decimal places when appropriate.

7. $10^{1/3}$ 8. $\sqrt[6]{13}$ 9. $21^{-1/4}$ 10. $(\sqrt[4]{81})^{-2}$

Example 3 *Solve equations using nth roots*

a. $2x^6 = 1458$

$x^6 =$ ______

$x =$ ______

$x =$ ______

b. $(x + 4)^3 = 12$

$x + 4 =$ ______

$x =$ ______

$x \approx$ ______

Solve the equation. Round the result to two decimal places when appropriate.

11. $-6x^2 = -180$ 12. $x^3 - 9 = 31$ 13. $(x + 8)^4 = 2$

Example 4 *Use nth roots in problem solving*

Animal Population The population P of a certain animal species after t months can be modeled by $P = C(1.21)^{t/3}$ where C is the initial population. Find the population after 19 months if the initial population was 75.

Solution

$P = C(1.21)^{t/3}$	**Write model for population.**
$=$ ______	**Substitute for C and t.**
$\approx$ ______	**Use a calculator.**

The population of the species is about ______ after 19 months.

Assignment 6-1 Pages 417-418, #8-32 even, 38-41 all, 50-58 even, 60, 61

Name ______________________ Date ____________

LESSON 6.1

Practice B

For use with pages 414–419

Rewrite the expression using rational exponent notation.

1. $\sqrt[3]{7}$
2. $(\sqrt[3]{6})^2$
3. $(\sqrt[5]{14})^4$
4. $(\sqrt[7]{-21})^3$
5. $(\sqrt[8]{11})^7$
6. $(\sqrt[9]{-2})^4$

Rewrite the expression using radical notation.

7. $17^{1/3}$
8. $44^{1/6}$
9. $33^{2/3}$
10. $9^{5/3}$
11. $(-28)^{7/5}$
12. $39^{4/7}$

Evaluate the expression without using a calculator.

13. $(\sqrt[3]{8})^2$
14. $(\sqrt[4]{16})^3$
15. $(\sqrt[4]{81})^4$
16. $36^{3/2}$
17. $4^{5/2}$
18. $27^{2/3}$
19. $125^{4/3}$
20. $(-8)^{1/3}$
21. $(-32)^{3/5}$

Evaluate the expression using a calculator. Round the result to two decimal places when appropriate.

22. $\sqrt[3]{38}$
23. $\sqrt[6]{112}$
24. $\sqrt[7]{-215}$
25. $(241)^{1/5}$
26. $(-133)^{1/3}$
27. $(69)^{1/4}$
28. $(96)^{2/3}$
29. $(356)^{5/9}$
30. $(-2427)^{4/7}$

31. **Geometry** Find the radius of a sphere with a volume of 589 cubic centimeters.

$V = \frac{4}{3}\pi r^3$

Solve the equation. Round the result to **two decimal places** when appropriate.

32. $x^3 + 17 = 132$
33. $2x^5 + 73 = 53$
34. $(x + 3)^4 = 362$

Algebra II

Name ______________________________

Date ____________________ Hr _____

Pre Section 6.2

	2nd Power	3rd Power	4th Power	5th Power
1	1^2	1^3	1^4	1^5
2	2^2	2^3	2^4	2^5
3	3^2	3^3	3^4	3^5
4	4^2	4^3	4^4	4^5
5	5^2	5^3	5^4	
6	6^2	6^3		
7	7^2	7^3		
8	8^2			
9	9^2			
10	10^2			
11	11^2			
12	12^2			

$3 = 3$

$\sqrt{3} = 3$

$\sqrt[3]{3} = 3$

$\sqrt[4]{3} = 3$

Algebra II

Worksheet

Name ______________________________

Date ______________________ Hr ____

1) $\sqrt{x^2} =$

2) $\sqrt[3]{x^3} =$

3) $\sqrt[4]{x^4} =$

4) $\sqrt{9} =$

5) $\sqrt[3]{27} =$

6) $\sqrt[4]{81} =$

7) $\sqrt[3]{x^3 y^4 z^5}$

8) $\sqrt[4]{x^4 y^6}$

9) $\sqrt{x^5 y^7}$

10) $\sqrt{16 x^2 y^4 z}$

Advanced Algebra

Name______________________
Date______________ Hr________

Section 6.2, Apply Properties of Rational Exponents

GOAL **Simplify expressions involving rational exponents.**

Vocabulary

A radical with index n is in **simplest form** if the radicand has no perfect nth powers as factors and any denominator has been rationalized.

Two radical expressions with the same index and radicand are **like radicals.**

PROPERTIES OF RATIONAL EXPONENTS

Let a and b be real numbers and let m and n be rational numbers. The following properties have the same names as those in Lesson 5.1, but now apply to rational exponents.

Property

1. $a^m \cdot a^n = a^{m+n}$ $\qquad 4^{1/2} \cdot 4^{3/2} = 4^{(1/2 + 3/2)}$ ____________

2. $(a^m)^n = a^{mn}$ $\qquad (2^{5/2})^2 = 2^{(5/2 \cdot 2)}$ ____________

3. $(ab)^m = a^m b^m$ $\qquad (16 \cdot 4)^{1/2} = 16^{1/2} \cdot 4^{1/2}$ ____________

4. $a^{-m} = \frac{1}{a^m}, a \neq 0$ $\qquad 25^{-1/2} = \frac{1}{25^{1/2}} =$ ____

5. $\frac{a^m}{a^n} = a^{m-n}, a \neq 0$ $\qquad \frac{3^{5/2}}{3^{1/2}} = 3^{(5/2 - 1/2)} =$ ____________

6. $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}, b \neq 0$ $\qquad \left(\frac{27}{8}\right)^{1/3} = \frac{27^{1/3}}{8^{1/3}} =$ ____

Example 1 *Use properties of exponents*

Use the properties of rational exponents to simplify the expression.

a. $9^{1/2} \cdot 9^{3/4} =$ ______________

b. $(7^{2/3} \cdot 5^{1/6})^3 =$ ______________

$=$ ______________

$=$ ______________

c. $\frac{3^{5/6}}{3^{1/3}} =$ ______________

d. $\left(\frac{16^{2/3}}{4^{2/3}}\right)^4 =$ ______________

PROPERTIES OF RADICALS

Product Property of Radicals

$\sqrt[n]{a \cdot b} =$ ______________

Quotient Property of Radicals

$\sqrt[n]{\frac{a}{b}} = \frac{}{}$, $b \neq 0$

Example 2 *Use properties of radicals*

Use the properties of radicals to simplify the expression.

a. $\sqrt[5]{27} \cdot \sqrt[5]{9} =$ ______ $=$ ______ $=$ ___ **Product property**

b. $\frac{\sqrt[3]{192}}{\sqrt[3]{3}} = \frac{}{}$ $=$ ______ $=$ ___ **Quotient property**

Example 3 *Write radicals in simplest form*

Write the expression in simplest form.

$\sqrt[5]{128} =$ ______ **Factor out perfect fifth power.**

$=$ ______ $\cdot$ ______ **Product property**

$=$ ______ **Simplify.**

Example 4 ***Add and subtract like radicals and roots***

Simplify the expression.

a. $2(12^{2/3}) + 7(12^{2/3}) =$ ______________

b. $\sqrt[4]{48} - \sqrt[4]{3} =$ ______ $\cdot$ ______ $-$ ______

$=$ ______________

Simplify the expression.

1. $(8^{1/2} \cdot 9^{1/4})^2$ **2.** $\left(\frac{10}{10^{2/3}}\right)^2$ **3.** $125^{-1/3}$ **4.** $11^{-1/2} \cdot 11^{5/2}$

5. $\sqrt{18} \cdot \sqrt{27}$ **6.** $\sqrt[3]{16} \cdot \sqrt[3]{24}$ **7.** $\frac{\sqrt{200}}{\sqrt{8}}$ **8.** $\frac{\sqrt[5]{160}}{\sqrt[5]{5}}$

9. $\sqrt[3]{432}$ **10.** $\frac{\sqrt[4]{2}}{\sqrt[4]{3}}$ **11.** $3(6)^{1/4} + 5(6)^{1/4}$ **12.** $9(54)^{1/2} - 54^{1/2}$

Example 5 *Simplify expressions involving variables*

Simplify the expression. Assume all variables are positive.

a. $\sqrt[5]{32x^{15}} =$ ______

b. $(36m^4n^{10})^{1/2} =$ ______

$=$ ______

c. $\sqrt[3]{\frac{a^9}{b^6}} =$ ______

d. $\frac{42x^4z^7}{6x^{3/2}y^{-3}z^5} =$ ______

Example 6 *Write variable expressions in simplest form*

Write the expression in simplest form. Assume all variables are positive.

$\sqrt[4]{\frac{a^2}{b^6}} =$ ______ Make denominator a perfect fourth power.

$=$ ______ Simplify.

$=$ ______

Example 7 *Add and subtract expressions involving variables*

Perform the indicated operation. Assume all variables are positive.

a. $10\sqrt[5]{y} - 6\sqrt[5]{y} =$ ______

b. $3a^2b^{1/4} + 4a^2b^{1/4} =$ ______

Simplify the expression. Assume all variables are positive.

13. $\sqrt[4]{\frac{81m^4}{16n^8}}$

14. $(8p^6q^3)^{2/3}$

15. $\sqrt[3]{\frac{2x}{(3y)^2}}$

16. $6\sqrt{h} - 8\sqrt{h}$

17. $11a^{1/2}b - 4a^{1/2}b$

18. $3\sqrt{5y^4} - y\sqrt{20y^2}$

Algebra 2B **Section 6-2**

Variable Expressions: The properties of rational exponents and radicals can also be applied to expressions involving variables. Because a variable can be positive, negative, or zero, sometimes absolute value is needed when simplifying a variable expression.

	Rule	**Example**
When *n* is odd	$\sqrt[n]{x^n} = x$	
When *n* is even	$\sqrt[n]{x^n} = \lvert x \rvert$	

Absolute value is not needed when all variables are assumed to be positive.

1) **VOCABULARY** Are $2\sqrt{5}$ and $2\sqrt[3]{5}$ like radicals?

Simplify the expression (rational exponents).

2) $5^{3/2} \cdot 5^{1/2}$

3) $(6^{2/3})^{1/2}$

4) $3^{1/4} \cdot 27^{1/4}$

5) $\frac{9}{9^{-4/5}}$

6) $\left(\frac{7^3}{4^3}\right)^{-1/3}$

7) $(12^{3/5} \cdot 8^{3/5})^5$

8) $\frac{64^{5/9} \cdot 64^{2/9}}{4^{3/4}}$

9) $\frac{13^{3/7}}{13^{5/7}}$

Simplify the expression (radicals).

10) $\sqrt[3]{16} \cdot \sqrt[3]{4}$

11) $\sqrt[4]{8} \cdot \sqrt[4]{8}$

12) $\dfrac{\sqrt[5]{64}}{\sqrt[5]{2}}$

13) $\sqrt{20} \cdot \sqrt{5}$

Simplify the expression by combining radicals and roots.

14) $2\sqrt[6]{3} + 7\sqrt[6]{3}$

15) $\frac{3}{5}\sqrt[3]{5} - \frac{1}{5}\sqrt[3]{5}$

16) $25\sqrt[5]{2} - 15\sqrt[5]{2}$

17) $\frac{1}{8}\sqrt[4]{7} + \frac{3}{8}\sqrt[4]{7}$

18) $6\sqrt[3]{5} + 4\sqrt[3]{625}$

Simplify the variable expression. Assume all variables are positive.

19) $x^{1/4} \cdot x^{1/3}$

20) $(y^4)^{1/6}$

21) $\sqrt[4]{81x^4}$

22) $\dfrac{2}{x^{-3/2}}$

23) $\sqrt[3]{\dfrac{x^{15}}{y^6}}$

24) $\left(\sqrt[3]{x^2} \cdot \sqrt[6]{x^4}\right)^{-3}$

Write the expression in simplest form. Assume all variables are positive.

25) $\sqrt{49x^5}$

26) $\sqrt{x^2yz^3} \cdot \sqrt{x^3z^5}$

27) $\sqrt[4]{x^6}$

Combine variable expressions. Assume all variables are positive.

28) $3\sqrt[5]{x} + 9\sqrt[5]{x}$

29) $\frac{3}{4}y^{3/2} - \frac{1}{4}y^{3/2}$

30) $-7\sqrt[3]{y} + 16\sqrt[3]{y}$

31) $(x^4y)^{1/2} + (xy^{1/4})^2$

32) $x\sqrt{9x^3} - 2\sqrt{x^5}$

Irrational Exponents: Simplify the expression. Assume all variables are positive.

33) $\frac{x^{5\sqrt{3}}}{x^{2\sqrt{3}}}$

34) $(x^{\sqrt{2}})^{\sqrt{3}}$

35) $x^2y^{\sqrt{2}} + 3x^2y^{\sqrt{2}}$

36) **Circumference** The equatorial circumference of the Moon is 1.09×10^4 kilometers. One kilometer is equivalent to 3.94×10^4 inches. What is the equatorial circumference of the Moon in inches?

37) **Bowling Ball** A bowling ball is submerged in a tub of water. As a result, a total of 333 cubic inches of water is displaced. Use the formula for the volume of a sphere to find the radius of the bowling ball.

Name ______________________ Date ____________

LESSON 6.2

Practice B

For use with pages 420–427

Simplify the expression using the properties of radicals and rational exponents.

1. $7^{1/3} \cdot 7^{4/3}$
2. $\dfrac{4^{2/3}}{4^{1/3}}$
3. $(6^{2/3})^{3/4}$
4. $5^{1/4} \cdot 3^{1/4}$
5. $\sqrt[4]{2} \cdot \sqrt[4]{8}$
6. $\dfrac{\sqrt[4]{192}}{\sqrt[4]{6}}$
7. $\dfrac{11}{\sqrt[4]{11}}$
8. $\sqrt[3]{7} \cdot \sqrt[3]{49}$
9. $(3^{3/2})^2$
10. $\left(\dfrac{54}{64}\right)^{1/3}$
11. $\dfrac{\sqrt[4]{32}}{\sqrt[4]{2}}$
12. $\dfrac{\sqrt[5]{5}}{\sqrt[5]{27}}$

Simplify the expression. Assume all variables are positive.

13. $x^{5/3} \cdot x^{4/3}$
14. $\sqrt{x^{2/5}}$
15. $(x^{1/2})^{2/7}$
16. $\left(\dfrac{x^2}{27}\right)^{1/3}$
17. $\sqrt[3]{16x^4}$
18. $(x^{-3})^{2/5}$
19. $\dfrac{x^{7/5}}{x^{4/5}}$
20. $\dfrac{\sqrt[3]{64x^3y}}{4x^{-3}y}$
21. $x^5 \cdot x^{\sqrt{3}}$
22. $(x^{\sqrt{2}})^{3\sqrt{2}}$
23. $\dfrac{x^{4\sqrt{3}}}{2x^{2\sqrt{3}}}$
24. $(\sqrt[3]{x^4} \cdot \sqrt{x^5})^{-2}$

Perform the indicated operation. Assume all variables are positive.

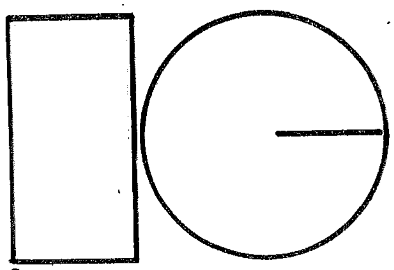
25. $6\sqrt[3]{5} + 2\sqrt[3]{5}$
26. $5\sqrt{5} - \sqrt{45}$
27. $2\sqrt{27} - 3\sqrt{48}$
28. $2\sqrt{x} + 7\sqrt{x}$
29. $3(x^{1/2}y^3)^2 - (x^3y^{18})^{1/3}$
30. $4x^{\sqrt{3}} + x^{\sqrt{3}}$

Write the expression in simplest form. Assume all variables are positive.

31. $\sqrt[4]{3x^7y^9z^3}$
32. $\sqrt{x^3y^4z} \cdot \sqrt{xyz^4}$
33. $\sqrt[3]{\dfrac{81x^4y^7}{8xy^4}}$

34. **Circumference** The equatorial circumference of Earth is 4.01×10^4 kilometers. One kilometer is equivalent to 3.94×10^4 inches. What is the equatorial circumference of Earth in inches?

35. **Swimming Pool** A wooden deck and a circular swimming pool cover an area of 514.16 square feet of the lawn. The rectangular deck is 20 feet wide and 10 feet long. What is the radius of the pool?



Algebra II **Name** ____________________

Quick Review **Date** ______________ **Hour** ____

Section 6.2 Properties of Rational Exponents

Properties of Rational Exponents

1. $a^m \cdot a^n =$

   $3^{1/2} \cdot 3^{3/2} =$

2. $(a^m)^n =$

   $(4^{3/2})^2 =$

3. $(ab)^m =$

   $(9 \cdot 4)^{1/2} =$

4. $a^{-m} =$

   $25^{-1/2} =$

5. $\dfrac{a^m}{a^n} =$

   $\dfrac{6^{5/2}}{6^{1/2}} =$

6. $\left(\dfrac{a}{b}\right)^m =$

   $\left(\dfrac{8}{27}\right)^{1/3} =$

Advanced Algebra

Name______________________
Date______________Hr________

Section 6.3, Perform Function Operations and Composition

GOAL **Perform operations with functions.**

Vocabulary

A **power function** has the form $y = ax^b$ where a is a real number and b is a rational number.

The **composition** of a function g with a function f is: $h(x) = g(f(x))$ where the domain of h is the set of all x-values such that x is in the domain of f and $f(x)$ is in the domain of g.

OPERATIONS ON FUNCTIONS

Let f and g be any two functions. A new function h can be defined by performing any of the four basic operations on f and g.

Operation and Definition	Example: $f(x) = 3x$, $g(x) = x + 3$
Addition $h(x) = f(x) + g(x)$	$h(x) = 3x + (x + 3)$ $=$ ________
Subtraction $h(x) = f(x) - g(x)$	$h(x) = 3x - (x + 3)$ $=$ ________
Multiplication $h(x) = f(x) \cdot g(x)$	$h(x) = 3x(x + 3)$ $=$ __________
Division $h(x) = \frac{f(x)}{g(x)}$	$h(x) = \frac{\_\_\_\_}{\_\_\_\_}$

The domain of h consists of the x-values that are in the domains of _____________. Additionally, the domain of a quotient does not include x-values for which $g(x) =$ ____.

Suppose the function $p(x) = -0.02x^2 + 45x - 4000$ gives the profit that a store obtains by selling x bicycles in one month. You can substitute various values or expressions for x in order to find the profit. Here are some examples.

$$p(600) = -0.02(600)^2 + 45(600) - 4000 = 15{,}800$$

$$p(t) = -0.02t^2 + 45t - 4000$$

$$p(x + 10) = -0.02(x + 10)^2 + 45(x + 10) - 4000$$
$$= -0.02t^2 + 44.6x - 3552$$

The function $f(x) = 14x$ gives the total cost of buying x CDs at \$14 each. Evaluate each of the following.

1. $f(3)$
2. $f(t)$
3. $f(3s)$
4. $f(2x + 5)$

The function $g(x) = \dfrac{300}{x}$ gives the number of hours required to drive 300 miles at a speed of x miles per hour. Evaluate each of the following.

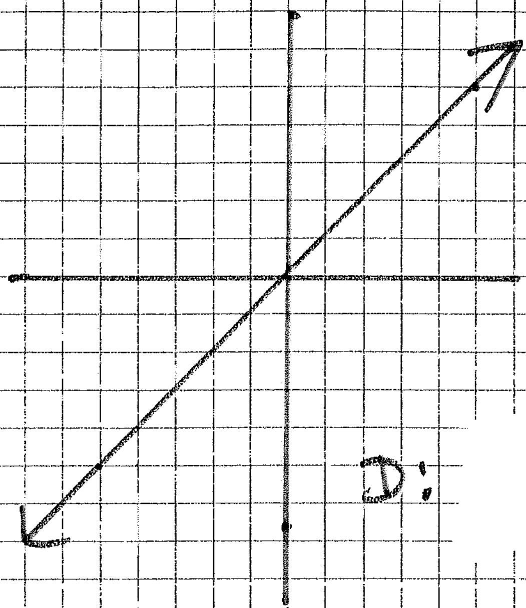
5. $g(50)$
6. $g(r)$
7. $g(20p)$
8. $g(75 - x)$

The function $h(t) = -16t^2 + 48t$ gives the height in feet after t seconds of a projectile fired upward from ground level at a speed of 48 feet per second. Evaluate each of the following.

9. $h(2.5)$
10. $h(x)$
11. $h(3x)$
12. $h(t + 1)$

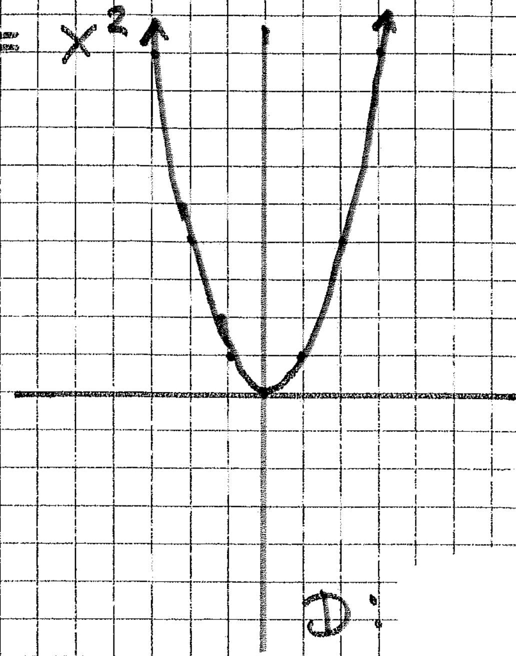
Domain

$y = x$



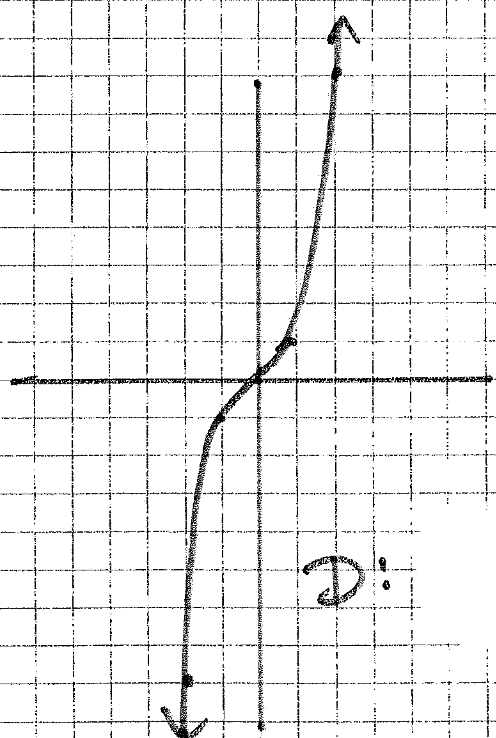
D:

$y = x^2$



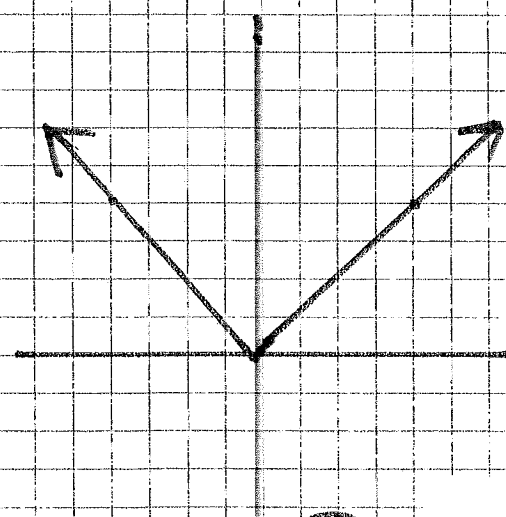
D:

$y = x^3$



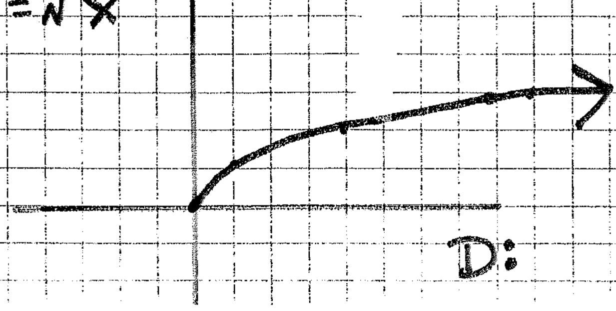
D:

$y = |x|$



D:

$y = \sqrt{x}$



D:

6-3

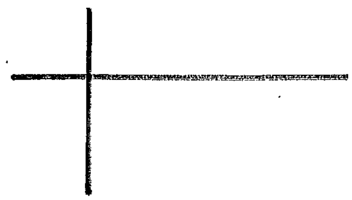
Example 1 *Add and subtract functions*

Let $f(x) = 3x^{1/2}$ and $g(x) = -5x^{1/2}$. Find the following.

a. $f(x) + g(x)$

b. $f(x) - g(x)$

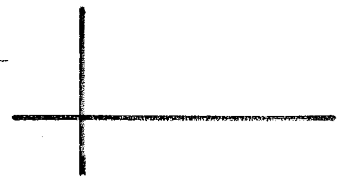
c. the domains of $f + g$ and $f - g$



Solution

a. $f(x) + g(x) = 3x^{1/2} + (-5x^{1/2})$

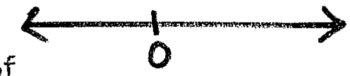
$= $ ________________



b. $f(x) - g(x) = 3x^{1/2} - (-5x^{1/2})$

$= $ ________________

c. The functions f and g each have the same domain: ________________. So, the domains of $f + g$ and $f - g$ also consist of ________________ ________.



Example 2 *Multiply and divide functions*

Let $f(x) = 7x$ and $g(x) = x^{1/6}$. Find the following.

a. $f(x) \cdot g(x)$

b. $\dfrac{f(x)}{g(x)}$

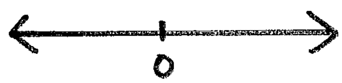
c. the domains of $f \cdot g$ and $\dfrac{f}{g}$

Solution

a. $f(x) \cdot g(x) = (7x)(x^{1/6}) = $ ________________

b. $\dfrac{f(x)}{g(x)} = $ ________________

c. The domain of f consists of ________________, and the domain of g consists of ________________ ________. So, the domain of $f \cdot g$ consists of ____ ________________. Because $g(0) = $ ____, the domain of $\dfrac{f}{g}$ is restricted to ________________ ________.



6-3 Domain

$7x$ $\quad 7x^2$ $\quad 7x^3$ $\quad 7x^4$ $\quad 3x^4-5x^3+2x^2-7$

D: D: D: D: D:

$x^{1/2}$ $\quad \frac{\sim}{x}$ $\quad \frac{\sim}{x-3}$ $\quad \frac{\sim}{x-8}$

D: D: D: D:

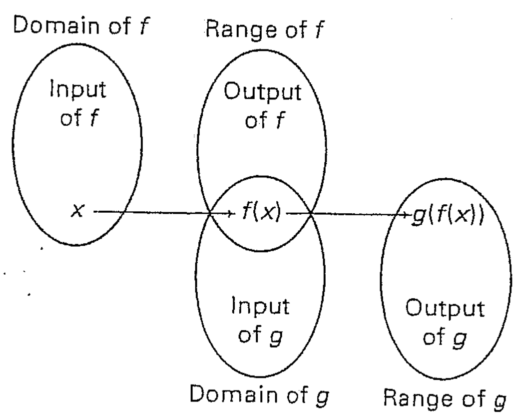
Let $f(x) = 2x^2$ and $g(x) = -3x^2$. Perform the indicated operation. State the domain.

1. $f(x) + g(x)$ **2.** $f(x) - g(x)$ **3.** $f(x) \cdot g(x)$ **4.** $\frac{f(x)}{g(x)}$

Domain: Domain: Domain: Domain:

COMPOSITION OF FUNCTIONS

The composition of a function g with a function f is $h(x) =$ ________. The domain of h is the set of all x-values such that x is in the domain of ___ and $f(x)$ is in the domain of ___.



Example 3 ***Find compositions of functions***

Let $f(x) = 6x^{-1}$ and $g(x) = 3x + 5$. Find the following.

a. $f(g(x))$ **b.** $g(f(x))$ **c.** $f(f(x))$

d. the domain of each composition

Solution

a. $f(g(x)) = f(3x + 5) =$ ______________

b. $g(f(x)) = g(6x^{-1})$

$=$ ______________

c. $f(f(x)) = f(6x^{-1}) =$ ______________

d. The domain of $f(g(x))$ consists of ______________

except $x =$ ____ because $g($____$) = 0$ is not in

the ______________. (Note that $f(0) =$ ____, which

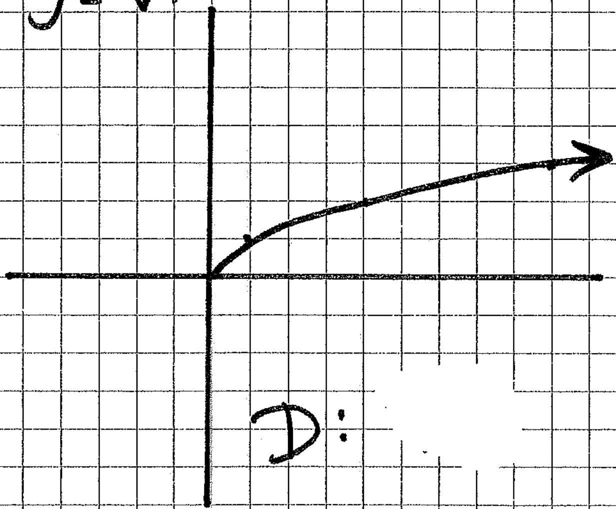
is ______________.) The domains of $g(f(x))$ and $f(f(x))$

consist of ______________ except $x =$ ____, again

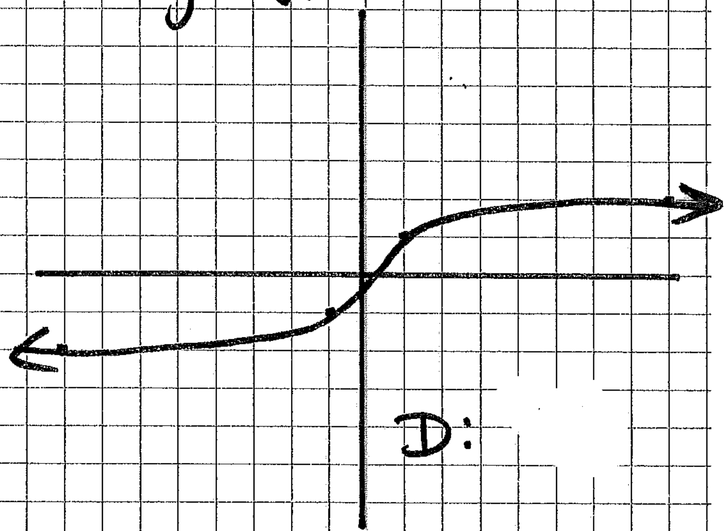
because ______________.

5. Let $f(x) = 3x^2$ and $g(x) = x^2 + 5$. Find (a) $f(g(x))$ and (b) $g(f(x))$.

$y = \sqrt{x}$



$y = \sqrt[3]{x}$



6-3
p. 432

$f(x) = 3x + 2$ $\quad$ $g(x) = -x^2$

20) $f(g(-3))$

$h(x) = \frac{x+4}{3}$ $\quad$ $f(x) = 3x^{-1}$

30) $h(f(x))$

4) $g(x) \quad + \quad f(x)$

$5x^{1/3} + 4x^{1/2}$ $\quad$ $-3x^{1/3} + 4x^{1/2}$

Sports Store You purchase water skis with a price tag of $180 dollars. The sports store applies a newspaper coupon of $50 and a 10% store discount. Use composition to find the final price of the purchase when the coupon is applied before the discount. Use composition to find the final price of the purchase when the discount is applied before the coupon.

STEP 1 **Write** functions for the discounts. Let x be the tag price, $f(x)$ be the price after the $50 coupon, and $g(x)$ be the price after the 10% store discount.

Function for $50 coupon: $f(x) = x - 50$

Function for 10% discount: $g(x) = x - 0.10x = 0.90x$

STEP 2 **Compose** the functions.

$50 coupon is applied first: $g(f(x)) = g(x - 50) = 0.90(x - 50)$

10% discount is applied first: $f(g(x)) = f(0.90x) = 0.90x - 50$

STEP 3 **Evaluate** the functions $g(f(x))$ and $f(g(x))$ when $x = 180$.

$g(f(180) = 0.90(180 - 50) = 0.90(130) = \$117$

$f(g(180) = 0.90(180) - 50) = 162 - 50 = \$112$

The final price is $117 when the $50 coupon is applied before the 10% discount. The final price is $112 when the 10% discount is applied before the $50 coupon.

6. Rework Example 4 for a price tag of $200, a $30 coupon, and a 20% discount.

Function for

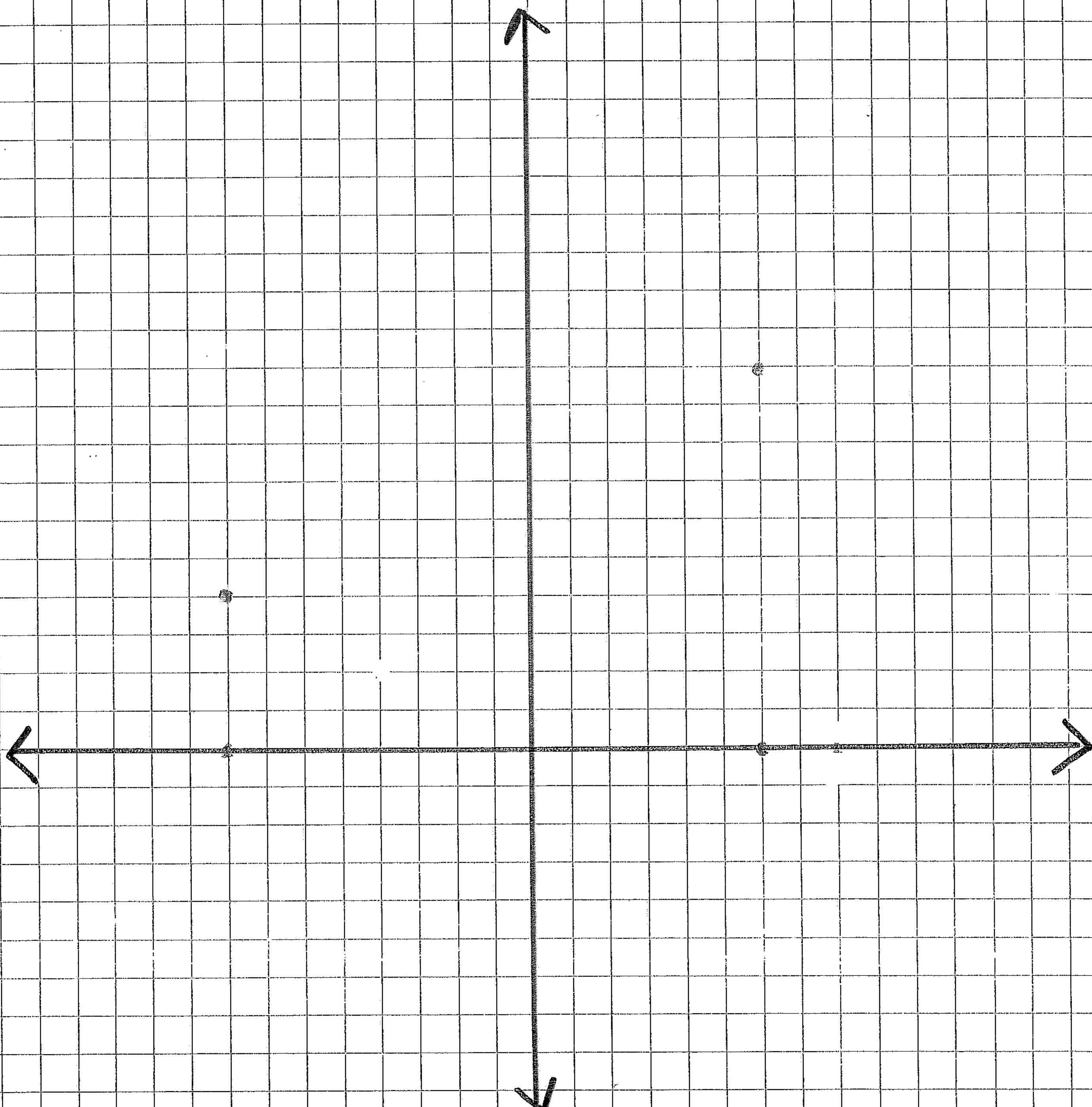
Function for

coupon applied first

discount applied first

Assignment 6-3 Pages 432-433, #4-34 even, 44

Sum of Functions ?



Algebra II

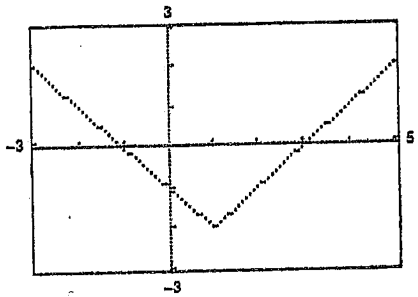
Name ____________________

Hour _______

Domain and Range

State the Domain and Range for each graph.

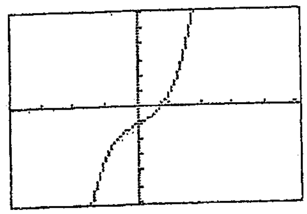
1)



Domain:

Range:

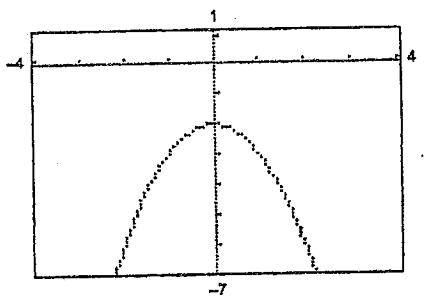
2)



Domain:

Range:

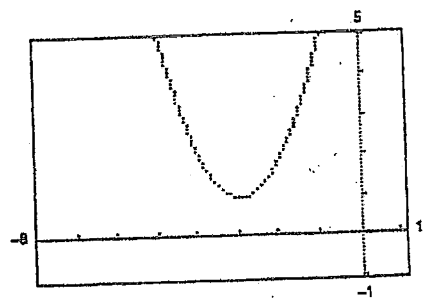
3)



Domain:

Range:

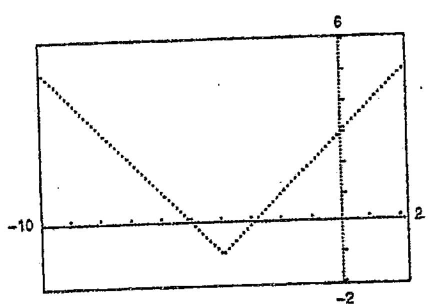
4)



Domain:

Range:

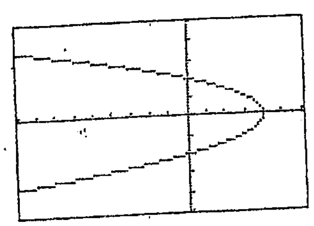
5)



Domain:

Range:

6)



Domain:

Range:

Algebra II Name ____________________

Worksheet 6-2 Practice A Date ______________ Hr ____

Simplify the expression using the properties of rational exponents.

1. $5^{1/2} \cdot 5^{3/2}$
2. $(3^{2/3})^{5/2}$
3. $(4 \cdot 3)^{2/3}$
4. $7^{-3/2}$
5. $\dfrac{9^{3/5}}{9^{2/5}}$
6. $\left(\dfrac{5}{4}\right)^{1/6}$

Simplify the expression using the properties of radicals.

7. $\sqrt{5} \cdot \sqrt{2}$
8. $\sqrt[3]{14} \cdot \sqrt[3]{196}$
9. $\dfrac{\sqrt{27}}{\sqrt{3}}$
10. $\sqrt{\dfrac{4}{9}}$
11. $\sqrt[4]{5} \cdot \sqrt[4]{2000}$
12. $\dfrac{\sqrt{10} \cdot \sqrt{21}}{\sqrt{15}}$

Simplify the expression. Assume all variables are positive.

13. $x^{1/3} \cdot x^{4/3}$
14. $(x^{2/5})^2$
15. $(x^{3/2})^{1/2}$
16. $(8x)^{1/3}$
17. $x^{-4/3}$
18. $(x^{5/6})^{-3}$
19. $\dfrac{x^{5/6}}{x^{1/6}}$
20. $\dfrac{x^{2/3}}{x^{5/3}}$
21. $\left(\dfrac{64}{x}\right)^{2/3}$

Perform the indicated operation. Assume all variables are positive.

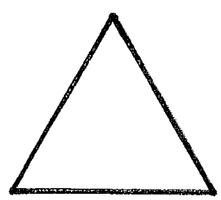
22. $\sqrt{5} + 3\sqrt{5}$
23. $6\sqrt{7} - 3\sqrt{7}$
24. $2\sqrt[5]{13} + 5\sqrt[5]{13}$
25. $7\sqrt{5} - 2\sqrt{20}$
26. $2\sqrt{27} + 4\sqrt{75}$
27. $\sqrt[3]{16} + \sqrt[3]{54}$
28. $3\sqrt{x} - 8\sqrt{x}$
29. $5\sqrt[3]{x} + 2\sqrt[3]{x}$
30. $-4\sqrt[4]{x} - 6\sqrt[4]{x}$

Write the expression in simplest form. Assume all variables are positive.

31. $\sqrt{64x^3}$
32. $\sqrt{\dfrac{x^3}{x^4}}$
33. $\sqrt[3]{x^3y^4z^5}$
34. $\sqrt[3]{8x^3y^6z^4}$
35. $\sqrt{xy^2z^3} + \sqrt{9xy^2z^3}$
36. $\sqrt[3]{x^3y^2z^7}$

37. **Geometry** The area of an equilateral triangle is given by $A = \dfrac{\sqrt{3}}{4}s^2$.

Find the length of the side s of an equilateral triangle with an area of $\sqrt{27}$ square units.



Name: ____________________ Class: ________________ Date: _________ ID: A

Algebra II 6-3 Worksheet

1. Let $f(x) = 16 - x^2$ and $g(x) = 4 - x$. Find $f(x) - g(x)$.

2. Let $f(x) = 1 - x^2$ and $g(x) = 1 - x$. Find $f(x) \cdot g(x)$.

3. Let $r(x) = 2x$ and $s(x) = x^3 - 3$. Find $s(r(2))$.

4. Let $r(x) = x^2 - 3$ and $s(x) = x^3 - 6$. Find $r(s(-1))$.

5. Let $g(x) = -5x^2$. Find $g(g(-1))$.

6. Let $f(x) = x^2 - 5$ and $g(x) = 3x^2$. Find $g(f(x))$.

7. Let $f(x) = x^2 - 4$ and $g(x) = -3x^2$. Find $f(g(x))$.

Algebra 2 Name________________ Date__________ Hr______

Quiz 6.1, 6.2 Review

1) Rewrite the expression using rational exponent notation

$\sqrt[6]{15}$

2) Rewrite the expression using radical notation.

$8^{1/4}$

3) Evaluate the expression without a calculator.

$\sqrt[5]{32}$

4) Evaluate the expression using a calculator. Round to 2 decimal places.

$\sqrt[3]{18}$

5) Simplify the expression.

$(3^{2/3})^{5/2}$

6) Simplify the expression.

$\dfrac{\sqrt{10} \cdot \sqrt{21}}{\sqrt{15}}$

7) Simplify the expression.

$(x^{3/2})^{1/2}$

8) Perform the indicated operation.

$6\sqrt{7} - 3\sqrt{7}$

9) Write the expression in simplest form.

$\sqrt[3]{8x^3y^6z^4}$

10) Write the expression in simplest form.

$\sqrt[3]{x^3y^2z^7}$

11) Simplify the expression

$x^{5/3} \cdot x^{4/3}$

12) Simplify the expression.

$\dfrac{x^{7/5}}{x^{4/5}}$

Advanced Algebra

Name________________________

Date________________Hr________

Section 6.4, Use Inverse Functions

GOAL Find inverse functions.

Vocabulary

An **inverse relation** interchanges the input and output values of the original relation.

Functions f and g are **inverse functions** of each other provided $f(g(x)) = x$ and $g(f(x)) = x$. The function g is denoted by f^{-1}, read as "f inverse."

Example 1 *Find an inverse relation*

Find an equation for the inverse of the relation $y = 7x - 4$.

$y = 7x - 4$ **Write original equation.**

______________ **Switch x and y.**

______________ **Add ___ to each side.**

Solve for y. This is the inverse relation.

____

INVERSE FUNCTIONS

Functions f and g are inverses of each other provided:

$f(g(x)) =$ ___ and $g(f(x)) =$ ___

The function g is denoted by f^{-1}, read as "f inverse."

Example 2 *Verify that functions are inverses*

Verify that $f(x) = 7x - 4$ and $f^{-1}(x) = \frac{1}{7}x + \frac{4}{7}$ are inverses.

Show that $f(f^{-1}(x)) = x$.

$f(f^{-1}(x)) = f\left(\frac{1}{7}x + \frac{4}{7}\right)$

= ______________

= ______________

= ___

Show that $f^{-1}(f(x)) = x$.

$f^{-1}(f(x)) = f^{-1}(7x - 4)$

= ______________

= ______________

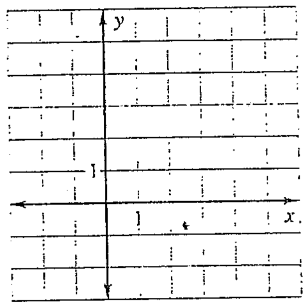
= ___

Example 3 *Find the inverse of a power function*

Find the inverse of $f(x) = 4x^2$, $x \le 0$. Then graph f and f^{-1}.

$f(x) = 4x^2$	**Write original function.**
$y = 4x^2$	**Replace $f(x)$ with y.**
________	**Switch x and y.**
________	**Divide each side by 4.**
________	**Take square roots of each side.**

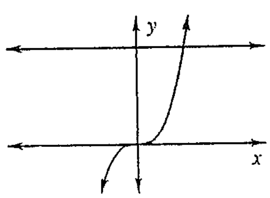
The domain of f is restricted to negative values of x. So, the range of f^{-1} must also be restricted to negative values, and therefore the inverse is $f^{-1}(x) =$ ________ . (If the domain were restricted to $x \ge 0$, you would choose $f^{-1}(x) =$ ________ .)



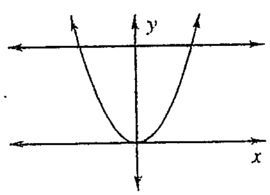
HORIZONTAL LINE TEST

The inverse of a function f is also a function if and only if no horizontal line intersects the graph of f ________ ________.

Function



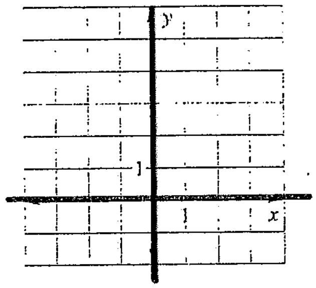
Not a function



Example 4 *Find the inverse of a cubic function*

Consider the function $f(x) = \frac{1}{4}x^3 + 3$. Determine whether the inverse of f is a function. Then find the inverse.

Solution



Graph the function f. Notice that no ______________ intersects the graph more than once. So, the inverse of f is itself a __________. To find an equation for f^{-1}, complete the following steps.

$f(x) = \frac{1}{4}x^3 + 3$ **Write original function.**

$y = \frac{1}{4}x^3 + 3$ **Replace $f(x)$ with y.**

______________ **Switch x and y.**

______________ **Subtract ____ from each side.**

______________ **Multiply each side by ____.**

______________ **Take cube root of each side.**

The inverse of f is $f^{-1}(x) =$ ____________.

Find $g(x)$, the inverse of the function. Verify that $f(x)$ and $g(x)$ are inverse functions.

1. $f(x) = -5x + 2$

2. $f(x) = 2x^4 - 3, x \geq 0$

$f(g(x))$

$f(g(x))$

$g(f(x))$

$g(f(x))$

Find $g(x)$, the inverse of the function. Verify that $f(x)$ and $g(x)$ are inverse functions.

3. $f(x) = 4x^3 - 5$

$f(g(x)) =$

$g(f(x)) =$

4. $f(x) = 3x^{0.25}$

$f(g(x)) =$

$g(f(x)) =$

5. Sales S (in thousands of dollars) of a product are modeled by $S = 6t^{1.5}$ where t is the number of months of production. Find the inverse model that gives time t as a function of sales S.

6. Use the inverse power model from Exercise 5 to estimate the number of months t when Sales S is \$314,300.

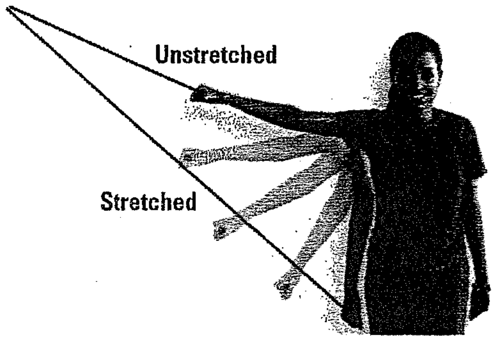
Assignment 6-4 Pages 442-444, #4-12 even, 16, 20, 22, 24, 26, 30-36 even (just graph in calculator), 38-44 even, 46, 47

6-4 Inverse Functions

EXAMPLE 3 Solve a multi-step problem

FITNESS Elastic bands can be used in exercising to provide a range of resistance. A band's resistance R (in pounds) can be modeled by $R = \frac{3}{8}L - 5$ where L is the total length of the stretched band (in inches).

- Find the inverse of the model.



Solution

STEP 1 **Find** the inverse function.

$R = \frac{3}{8}L - 5$ **Write original model.**

Add 5 to each side.

Multiply each side by $\frac{8}{3}$.

- Use the inverse function to find the length at which the band provides 19 pounds of resistance.

STEP 2 **Evaluate** the inverse function when $R = 19$.

▸ The band provides 19 pounds of resistance when it is stretched to inches.

6-4 Inverse Functions

1) Consider the function $f(x) = 3x^5 - 2$. Find the inverse.

2) Three operations are done to x to obtain the expression $2x^3 + 1$.

Raise x to the third power (cube it), multiply by 2, and add 1.

What three operations can be done to get back to x, and in what order?

3) You earn \$8 per hour, plus \$.20 for each unit x you produce per hour. The linear model for your hourly wage w in terms of x units produced is $w = 8 + 0.2x$. Find the inverse of the model.

Then use the inverse function to find the number of units required per hour for your hourly wage to be \$12.

Your wage is \$12 per hour when you produce ______ units.